FEDERAL UNIVERSITY OF CARIRI
CENTRE FOR AGRARIAN SCIENCES AND BIODIVERSITY
POSTGRADUATE PROGRAM IN SUSTAINABLE REGIONAL DEVELOPMENT

MODELING AND FORECASTING SMALL NATIONAL ECO-EFFICIENCY TIME SERIES

MANOEL ALEXANDRE DE LUCENA

CRATO
2024

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Dissertation presented to Postgraduate Program in Sustainable Regional Development, Centre for Agrarian Sciences and Biodiversity, Federal University of Cariri, as a partial requirement for obtaining a Master degree in Sustainable Regional Development.

Advisor: Dr. Paulo Renato Alves Firmino

CRATO

2024

Dados Internacionais de Catalogação na Publicação
Universidade Federal do Cariri
Sistema de Bibliotecas

L935m Lucena, Manoel Alexandre de.
Modeling and forecasting small national eco-efficiency time series / Manoel Alexandre de Lucena - 2024.
132 f. il. color.; 30 cm.
(Inclui bibliografia, p. 88-101).

Dissertação (Mestrado) – Universidade Federal do Cariri, Programa de Pós-graduação em Desenvolvimento Regional Sustentável (PRODER), Crato, 2024.
Trabalho escrito em língua inglesa.

Orientador: Prof. Dr. Paulo Renato Alves Firmino .

1. Sustainable development. 2. Environmental and economic impacts. 3. Machine learning algorithms. I. Firmino, Paulo Renato Alves - orientador. III. Título.

CDD 333.7

Bibliotecária: Maria Eliziana Pereira de Sousa – CRB 15/564

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Approval date: July 26, 2024

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ACKNOWLEDGEMENT

To God, the architect of the universe.

To my family for their support, especially my mother Maria Alexandre.

To my advisor Paulo Renato Alves Firmino, for your partner, patience, guidance and all the support during the master's degree.

To the examining board: Carlos Wagner and Margarita Robaina for accepting our invitation and for the several contributions to this document.

To Bárbara Salviano, for the support, encouragement and companionship.

To my colleagues on the course, especially Yara Eugenio, Welde, Esdras, Ramon, and Thais.

To the members of MESOR research group.

To the postgraduate program in Sustainable Regional Development at the Federal University of Cariri (UFCA).

To the Coordination Foundation for the Improvement of Higher Education Personnel (CAPES) for the financial support.

ABSTRACT

Eco-efficiency time series are useful for monitoring the relationship between economic and environmental variables. Thus, eco-efficiency forecasting provides savings in resources and time as well as support researchers and managers with insights into eco-efficiency in the future. It also allows monitoring environmental policy in different sectors of the economy. However, national eco-efficiency time series are mostly small or very small. Further, it is relevant to consider models that simultaneously involve all countries, that is, a pooled approach. Thus, the applied pooled approaches can verify whether just a pooled model can predict better than individual time series models for each country. In this context, this research aims to study a method for modeling and forecasting national eco-efficiency time series. To model the national eco-efficiency time series, two strategies are considered: (i) individual time series; and (ii) pooled approaches considering individual effects of each country and lags. Machine learning models for time series are adopted in both individual cases: Support Vector Regression (SVR), Long Short-Term Memory (LSTM), Decision Tree Regression (DTR); and ensemble: combination by Simple Average (SA), Simple Median (SM), Minimum Variance (MV); Random Forest Regression (RFR), and Extreme Gradient Boosting (XGB). Further, considering the individual approach, Autoregressive Integrated Moving Average (ARIMA) and Exponential Smoothing (ETS) are also considered. In turn, to obtain the national eco-efficiency time series, Data Envelopment Analysis combined with Window Analysis (WDEA) is applied. To calculate the window size in WDEA, a method based on the divergence of eco-efficiency is proposed. In particular, the ideal window width is the one that maximizes the eco-efficiency dispersion. The Mercosul, BRICS, and G18 countries were considered as case studies, involving annual eco-efficiency time series from 1995 to 2020. The ideal window size is equal to 1, with maximum dispersion of eco-efficiency between countries. In the three cases studied, the pooled approach won in 50% of the series in Mercosul, 25% in the BRICS and 9.1% in the G18. Particularly, of 19 best models, 12 (63.1%) were single models. The heterogeneity between the eco-efficiency series, given its importance in Window Analysis, may be unfavorable to the pooled approach. In addition, average eco-efficiency projected for 6-years-ahead was low. The results showed that for groups that are possibly more heterogeneous in terms of environmental and economic factors, the analysis of individual time series wins over the pooled approach. Given the low projected eco-efficiency, agreements and actions adapted to the reality of countries and groups can provide ways to increase the eco-efficiency of countries aligned with sustainable development. Therefore, considering these results, actions for policymakers can be proposed: (i) alignment of goals between countries and groups based on predicted eco-efficiency considering time series models; (ii) global agreement strategies that consider the individual reality of countries in terms of economic and environmental resource endowments; and (iii) use of technology to obtain and use sources of renewable resources that reduce greenhouse gas emissions.

Keywords: sustainable development; environmental and economic impacts; Window Data Envelopment Analysis; machine learning algorithms.

RESUMO

As séries temporais de ecoeficiência são úteis para monitorar a relação entre variáveis econômicas e ambientais. Assim, a previsão da ecoeficiência proporciona economia de recursos e tempo e fornece aos pesquisadores e gestores insights sobre a ecoeficiência no futuro e o monitoramento da política ambiental em diferentes setores da economia. Contudo, as séries cronológicas nacionais sobre a ecoeficiência são, na sua maioria, pequenas ou muito pequenas. Além disso, é relevante considerar modelos que envolvam simultaneamente todos os países, ou seja, uma abordagem empilhada. Assim, as abordagens agrupadas aplicadas podem verificar se apenas um modelo agrupado pode prever melhor do que modelos de séries temporais individuais para cada país. Neste contexto, esta pesquisa tem como objetivo estudar um método de modelagem e previsão de séries temporais de ecoeficiência nacional. Modelos individuais de aprendizado de máquina para séries temporais são adotados em ambos os casos: Support Vector Regression (SVR), Long Short-Term Memory (LSTM), Decision Tree Regression (DTR); e ensemble: combinação por Média Simples (SA), Mediana Simples (SM), Mínima Variância (MV); Random Forest Regression (RFR) e Extreme Gradient Boosting (XGB). Além disso, considerando a abordagem individual, também são consideradas a Autoregressive Integrated Moving Average (ARIMA) e Exponential Smoothing (ETS). Por sua vez, para obter a série temporal de ecoeficiência nacional, aplica-se a Análise Envoltória de Dados combinada com a Análise de Janela (WDEA). Para calcular o tamanho da janela no WDEA, é proposto um método baseado na divergência de ecoeficiência. Em particular, a largura ideal da janela é aquela que maximiza a dispersão da ecoeficiência. Os países Mercosul, BRICS e G18 foram considerados como estudos de caso, envolvendo séries temporais anuais de ecoeficiência de 1995 a 2020. Nos três casos estudados, a abordagem pooled venceu em 50% das séries no Mercosul, 25% nos BRICS e 9,1% no G18. Particularmente, dos 19 melhores modelos, 12 (63,1%)

foram modelos individuais. A heterogeneidade entre as séries de ecoeficiência, dada a sua importência em Análise de Janela, pode desfavorecer a abordagem pooled. Além disso, a ecoeficiência média projetada para os próximos 6 anos foi baixa. Os resultados mostraram que para grupos possivelmente mais heterogêneos em termos de fatores ambientais e econômicos, a análise de séries temporais individuais vence a abordagem agrupada. Dada a baixa ecoeficiência projetada, acordos e ações adaptadas à realidade dos países e grupos podem fornecer formas de aumentar a ecoeficiência de países alinhados ao desenvolvimento sustentável. Portanto, considerando estes resultados, podem ser propostas ações para os formuladores de políticas: (i) alinhamento de metas entre países e grupos com base na ecoeficiência prevista considerando modelos de séries temporais; (ii) estratégias de acordos globais que considerem a realidade individual dos países em termos de dotações de recursos económicos e ambientais; e (iii) uso de tecnologia para obtenção e utilização de fontes de recursos renováveis que reduzam as emissões de gases de efeito estufa.

Palavras-chaves: desenvolvimento sustentável; impactos ambientais e econômicos; Window Data Envelopment Analysis; algoritmos de aprendizado de máquina.

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LIST OF ABBREVIATIONS AND ACRONYMS

ACF	Autocorrelation Function
ANN	Artificial neural networks
AR	Autoregressive
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
BIC	Bayesian Information Criterion
CART	Classification And Regression Trees
CRS	Constant Returns to Scale
CV	cross-validation
DEA	Data Envelopment Analysis
DEA-CRS	Data Envelopment Analysis with Constant Returns to Scale
DMU	Decision Making Unit
DT	Decision Tree
DTR	Decision Tree Regression
ETS	Exponential smoothing
GDP	Gross Domestic Product
GM	Gray Model
GridSearch	Grid search
GridSearchCV	Grid search cross-validation
LSTM	Long Short-Term Memory
MA	Moving Average
MAD	Mean-Absolute Deviation
Mercosul	Common Market of the South
MPI	Malmquist Productivity Index
MV	Minimum Variance
PACF	Partial Autocorrelation Function

PARDL	Panel Autoregressive Distributed Lag
PVAR	Panel vector autoregressive
RandomizedSearch	Randomized search
RBFN	Radial basis function neural
RFR	Random Forest Regression
RMSE	Root Mean Squared Error
SA	Simple Average
SD	Standard Deviation
SDGs	Sustainable Development Goals
SES	Simple Exponential Smoothing
SFA	Stochastic Frontier Analysis
SM	Simple Median
SMA	Simple Moving Averages
SVM	Support Vector Machines
SVR	Support Vector Regression
UN	United Nations
WA	Window Analysis
WBCSD	World Business Council for Sustainable Development
WDEA	Window Data Envelopment Analysis
XGB	Extreme Gradient Boosting

LIST OF SYMBOLS

q	ARIMA model differentiation number
ϕ	Autoregressive model coefficient
$\overline{y}$	Average eco-efficiency
CO_2	Carbon dioxide
c	Constant of the regression model
y_o	Eco-efficiency score of the DMU o
δ, F, g	ETS coefficients
$h(t)$	Hidden layer vector
tanh	Hyperbolic tangent activation function
$K(.)$	Kernel function
kWh	Kilowatt-hours
α, α^*	Lagrange multipliers
$\mathbf{D}$	Matrix of dummies variables for the countries
$\mathbf{X}$	Matrix with p lags for the series
ε	Moving average model coefficient
μ	Node for tree based models
k	Number of DMUs
r	Number of inputs
m	Number of outputs
w	Number of windows
p	Order of the autoregressive process
d	Order of the moving averages process

γ	Parameter to kernel function
$\mathbf{Y}$	Pooled time series vector
C	Regularization constant of SVR model
$\mathbb{N}^*$	Set of non-zero natural numbers
σ	Sigmoid activation function
ξ_t^+, ξ_t^-	Slack variables of SVR model
ϵ	SVR model error tolerance
η	SVR model weight vector
N	Time series size
y_t	Time series values in t
u_i	Value observed for the input i
v_i	Value observed for the output i
ω	Weight assigned to the predictor in the combination by MV
n	Window size

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1 INTRODUCTION

Sustainable development has gained attention of local and global leaders, from the first, second, and third sectors. Brundtland Report defined sustainable development as the ability to meet the needs of the current generation without compromising future generations (WCED *et al.*, 1987; PEREIRA; MARTINS, 2021). For the effective application of sustainable development in response to various issues (e.g. industrialization, urbanization, environmental deterioration), social, economic, and environmental integration is necessary (MAJID *et al.*, 2023). To specifically assess the performance of territories or organizations in the promotion of economic investments and the respective environmental deterioration, eco-efficiency plays important role [1] (CASTILHO *et al.*, 2021). In this sense, eco-efficiency considers the economic and environmental dimensions. Particularly, eco-efficiency concerns the ability to produce more goods and services with less inputs and less impact on the environment, e.g. fewer consumption of natural resources and less pollution (CAMARERO *et al.*, 2014; SADORSKY, 2021). Furthermore, eco-efficiency can be quantified by the relationship between economic added value and environmental impacts (HELMINEN, 2000; KUOSMANEN; KORTELAINEN, 2005; PICAZO-TADEO *et al.*, 2012; MOUTINHO *et al.*, 2017).

Among the tools used to measure eco-efficiency in countries, the following stand out: Stochastic Frontier Analysis (SFA) and Data Envelopment Analysis (DEA) (CHARNES *et al.*, 1978). Particularly, DEA considers a set of units, called Decision Making Unit (DMU), in which each one uses inputs to produce outputs. In turn, to study eco-efficiency time series, Malmquist Productivity Index (MPI) (FÄRE *et al.*, 1994) and Window Analysis (WA) (CHARNES *et al.*, 1983) has been adopted. WA, based on the moving average principle, allows the analysis of

[1] In the literature, the terms “environmental efficiency” and “economic and environmental efficiency” are used as synonyms for eco-efficiency.

eco-efficiency time series from a window considering multiple DEA applications with DMU in different periods (COOPER *et al.*, 2006). Thus, in the WA application, the main objective is to determine the window size. Authors usually adopt different window sizes, observing different methods (ALKHARS *et al.*, 2022). In general, classical methods use information disregarding from eco-efficiency, i.e. time series size, number of inputs, number of outputs and number of DMUs. An exception is the method proposed by Hao *et al.* (2013), which considers the average deviation of eco-efficiency in each window in relation to the study period. However, alongside this literature based on average eco-efficiency, methods considering the dispersion of eco-efficiency may be promising. In this sense, this study contributing this discussion.

Considering this context, this study proposes a method for the ideal window size by analyzing the eco-efficiency convergence and divergence among DMUs. Based on economic theory, the convergence of eco-efficiency can be analyzed through σ-convergence and β-convergence (BAUMOL, 1986; BARRO; MARTIN, 1990; MARTIN, 1996). Particularly, σ-convergence examines the behavior of eco-efficiency scores applying dispersion metrics (e.g. variance and coefficient of variation) (NEŽINSKỲ, 2014; XU *et al.*, 2020). Therefore, we search for the window size that leads to the greatest eco-efficiency dispersion (i.e. divergence) among territories. Furthermore, in this study the use of WA combined with DEA is called Window Data Envelopment Analysis (WDEA). Thus, WDEA is applied to obtain eco-efficiency time series according to the window size. Therefore, eco-efficiency time series in this literature is measured by WDEA for sectoral, regional and national eco-efficiency (WANG; XIAO, 2017; LIN *et al.*, 2018; KÜPELİ *et al.*, 2019; KYRGIAKOS *et al.*, 2021; PISHGAR-KOMLEH *et al.*, 2021; HE *et al.*, 2021). This study considers national eco-efficiency time series.

Particularly, quantitative and computational modeling of eco-efficiency is important in the search for sustainable development (YU *et al.*, 2019). Wursthorn *et al.* (2011) highlights the importance of environmental-economic indicators, such as eco-efficiency, for monitoring environmental policy in different sectors of the economy. The adoption of such indicators allows the evaluation of countries in terms of environmental and economic variables. In this context, time series methods are important in modeling and forecasting eco-efficiency. According to the literature, eco-efficiency forecasting provides savings in resources and time and provides researchers and managers with insights into eco-efficiency in the future (WANG; XIAO, 2017). From this perspective, three types of approaches stand out: (i) to predict the series of inputs and outputs and then calculate the eco-efficiency (SADORSKY, 2021); (ii) to calculate eco-efficiency based on covariates (SONG *et al.*, 2013; WANG; XIAO, 2017; LI *et al.*, 2017; LIU *et al.*, 2017; MOUTINHO *et al.*, 2017; MOUTINHO *et al.*, 2020; XIA *et al.*, 2021; MOUTINHO; MADALENO, 2021b; HE *et al.*, 2021); (iii) to directly predict the eco-efficiency time series (SONG *et al.*, 2013; LI *et al.*, 2017; CARBONI; RUSSU, 2018; CASTILHO *et al.*, 2021; CHEN *et al.*, 2022). Approaches (i) and (ii) are not as promising. The first imposes the need to model distinct series, e.g. inputs and outputs. The second case requires a greater number of variables, as covariates are needed to calculate eco-efficiency and predict for new DMUs. Given these problems, this study considers the third case. The present research also works on the lack of specific studies for modeling and forecasting national eco-efficiency time series. In fact, for eco-efficiency time series, there is a lack of studies using more robust methods, such as machine learning combiners (ensemble).

Further, a characteristic of eco-efficiency time series forecasting studies is the size of the series. Mostly these are small time series (in some cases very small). The discussion about small time series forecasting present in the literature is incipient in eco-efficiency field. Although Song *et al.* (2013) apply Gray Model

(GM) to eco-efficiency small time series forecasting, however, no methods were found in the literature to specifically address the prediction of small eco-efficiency time series. In this research it is proposed the method for modeling and forecasting small eco-efficiency time series. The strategy is to pool the individual countries eco-efficiency time series. This structure is similar to the panel data approach (WOOLDRIDGE, 2010; HSIAO, 2022). However, in the proposed approach the data from the territories are pooled. Thus, the main advantage of this procedure is to increase the sample size, enlarging the degree of freedom of the models. Moreover, this structure adapts to the framework used in WDEA, in which DMUs (countries) are grouped. Thus, the idea is to train a general eco-efficiency model. Thus, the model obtained with the pooled time series can be compared with the models estimated for the individual series.

Therefore, in this work two methods are considered for the study of small time series of national eco-efficiency. Firstly, it is proposed by method for optimal window size in WDEA using eco-efficiency dispersion. This method makes it possible to analyze DMUs between windows by constructing eco-efficiency time series. Thus, this method contributes to the literature that discusses techniques for calculating window size in WDEA (ALKHARS *et al.*, 2022; LIN *et al.*, 2018; KYRGIAKOS *et al.*, 2021; PISHGAR-KOMLEH *et al.*, 2021). Next, it is studied for method modeling small eco-efficiency time series. The idea of pooling time series fundamentally contributes to increasing the sample. This approach contributes to the literature that studies models based on individual time series for eco-efficiency predictions (SONG *et al.*, 2013; LI *et al.*, 2017; MOUTINHO *et al.*, 2017; CARBONI; RUSSU, 2018; MOUTINHO *et al.*, 2020; XIA *et al.*, 2021; MOUTINHO; MADALENO, 2021b; HE *et al.*, 2021; CASTILHO *et al.*, 2021; CHEN *et al.*, 2022).

Thus, in addition to the literature, the research adopts this structure and applies single and ensemble models. Single time series machine learning models

(Autoregressive Integrated Moving Average (ARIMA), Exponential smoothing (ETS), Support Vector Regression (SVR), Long Short-Term Memory (LSTM), Decision Tree Regression (DTR)) are established formalisms for prediction. In the case of the ensemble, Simple Average (SA), Simple Median (SM), Minimum Variance (MV), Random Forest Regression (RFR) and Extreme Gradient Boosting (XGB) are applied. Particularly, ensemble models are at the forefront of forecasting approaches and have excelled in forecasting problems (GÉRON, 2022). Moreover, for applications of time series formalisms, three case studies are considered: Common Market of the South (Mercosul), BRICS, with five countries each, and G18 with 18 nations. Thus, 28 time series are studied from 1995 to 2020. Therefore, in analyzes of national eco-efficiency time series, the literature considers groups of countries.

1.1 Objectives

General objective

To study alternatives for modeling and forecasting national eco-efficiency (small) time series.

Specific objectives

(i) To obtain historical series of economic and environmental variables underlying national eco-efficiency;
(ii) To compute national eco-efficiency time series according to the optimal window size using data set from (i);
(iii) To model each individual national eco-efficiency time series from (ii) via single and ensemble predictors;
(iv) To compute pooled predictors for the pool of national eco-efficiency time

series from (ii) via single and ensemble predictors;

(v) To evaluate the predictors obtained from (iii) and (iv);

(vi) To predict national eco-efficiency based on the best models obtained in (v).

1.2 Dissertation structure

This research is organized as follows. Chapter 2 is devoted to the theoretical foundation. The proposed method to achieve the objectives is presented in Chapter 3. Chapter 4 brings the methodological procedure. In Chapter 5 three case studies on national eco-efficiency are presented (Mercosul, BRICS and G18). Moreover, modeling, forecasting and projection of time series in each case are carried out. Finally, Chapter 6 presents the research conclusions.

2 BACKGROUND

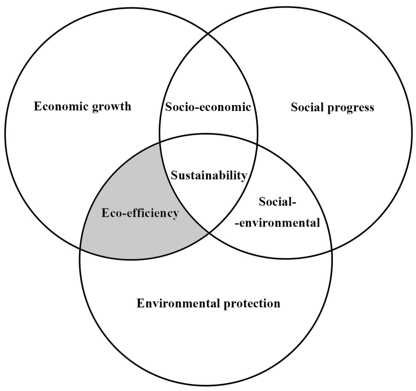
This chapter is dedicated to the theoretical foundation of this research. Firstly, the concepts of sustainable development, eco-efficiency, and time series forecasting are discussed. Then, DEA combined with WA (called WDEA), time series analysis and machine learning models are considered.

2.1 Sustainable development

In summary, the concept of sustainable development has emerged from environmental concerns. The idea is to reconcile two apparent conflicting paradigms: lasting economic growth and the protection of the environment and natural resources (HÁK *et al.*, 2016). Such concerns are made explicit in the book "The Limits to Growth", published in 1972 (MEADOWS *et al.*, 1972). The referred book deals prognostically with the impacts of economic and population growth, given finite natural resources. In line with such concerns, at the 1972 United Nations Conference on the Human Environment in Stockholm, the idea of sustainable development was treated internationally. Although this term was not explicitly mentioned at the conference, the international community agreed that development and the environment should be treated mutually (BASIAGO, 1996; MENSAH, 2019). This is the notion of sustainable development. Brundtland Report, published by WCED *et al.* (1987), defined sustainable development as the ability to meet the needs of the current generation without compromising the capabilities of future generations who will also meet their needs.

This definition, which can also be thought of within the scope of synchronic and diachronic solidarity, is aligned with intergenerational equity, which considers the short and long-term implications of sustainability and sustainable development (MENSAH, 2019). According to the literature, intergenerational equity is a commitment of the current generation to the future, implying that future generations have the right to enjoy nondeteriorated ecological and economic capacities (PADILLA, 2002). Thus, as sustainable development proposes a type of progress that does not compromise the resources of future generations, it can be understood from three perspectives. From this perspective, sustainable development can be thought of as: (i) an ecologically balanced development; (ii) an economically viable development; (iii) a socially responsible development (PEREIRA; MARTINS, 2021). The interaction between these dimensions (ecological, economic, and social) leads to important concepts. Particularly, this study is interested in the relationship between economic and environmental variables. From this relationship, we have the eco-efficiency concept, described in the next section. Figure 1 presents the dimensions of sustainable development with an emphasis on eco-efficiency.

Figure 1 – Intersections of the dimensions of sustainable development with emphasis on eco-efficiency



Source: adapted from Majid *et al.* (2023)

2.2 Eco-efficiency

The eco-efficiency concept was proposed by World Business Council for Sustainable Development (WBCSD). The concept emerged linked to corporate sustainability. In this context, eco-efficiency is only part of the corporate sustainable criteria (DYLLICK; HOCKERTS, 2002). For that reason, the term eco-efficiency corresponds to business activities that create economic value while decreasing ecological impact and resources use (DESIMONE; POPOFF, 2000). In other words, it refers to the ability to produce more goods and services with less consumption of natural resources (CAMARERO *et al.*, 2014; MIRMOZAFFARI *et al.*, 2020). Synthesizing several studies, eco-efficiency is defined as the ratio between the economic value added (result of economic activity) and environmental pressures (the impacts arising from economic activity in the environment) (HELMINEN, 2000; KUOSMANEN; KORTELAINEN, 2005; PICAZO-TADEO *et al.*, 2012). Based on these concepts, the eco-efficiency index is given by Equation (2.1):

$$Eco - efficiency = \frac{value\, added\, economic}{environmental\, pressures}, \tag{2.1}$$

is such a way that the better the result of the economic activity and the lesser the respective consumption of environmental resources, the greater the eco-efficiency is.

In general, eco-efficiency is an important tool in the discussion of environmental sustainability (SADORSKY, 2021). Although discussions were initially focused on companies, eco-efficiency analysis has expanded to different types of DMUs. From this perspective, the literature involves studies of eco-efficiency in companies, industry, and economy in general (CAMARERO *et al.*, 2013). Thus, analyzes of local, regional, and national eco-efficiency have been recurrent in the literature in recent years (SANTANA *et al.*, 2014; CAMIOTO *et al.*, 2016; CARBONI; RUSSU, 2018; WANG *et al.*, 2020; MOUTINHO; MADALENO, 2021b; SADORSKY, 2021; CHENG *et al.*, 2023). In particular, the present dissertation is dedicated to study national eco-efficiency. Therefore, the Subsection 2.2.1 summarizes the discussion on eco-efficiency measurement and forecasting.

2.2.1 Eco-efficiency measurement and forecasting

Empirical studies have been conducted by researchers in recent years aiming to measure environmental economic efficiency from different perspectives (CARBONI; RUSSU, 2018). Particularly, national eco-efficiency is measured utilizing variables as inputs and outputs (CAMIOTO *et al.*, 2016). Thus, in addition to environmental and economic variables, these studies employ proxies for capital, land, and labor (MOUTINHO; MADALENO, 2021b). The regional or national eco-efficiency metrics obtained by optimal combination between inputs and outputs, based on aggregate country data, such as, e.g., Gross Domestic Product (GDP) and greenhouse gas emissions (CAMIOTO *et al.*, 2016). Two types of tools are applied to measure eco-efficiency. One group applies parametric tools, which require an explicit equation between inputs and outputs, e.g. SFA. On the other hand, DEA is a non-parametric tool. DEA stood out to build eco-efficiency frontiers. Moreover, DEA uses linear programming methods for determined the optimal frontier with best practices observed in the frontier of reference. Particularly, in the analysis of eco-efficiency time series with DEA, two tools are employed: MPI and WA. MPI is used to obtain, in just two periods, relative efficiency and technological change in the transformation of inputs and outputs (FÄRE *et al.*, 1994). On the other hand, WA allows analyzing time series of efficiency considering time windows (CHARNES *et al.*, 1983). In this study, WA is considered.

Measuring eco-efficiency is important to assess, for example, levels of sustainable development. Thus, given its relationship with the environment, it is important to monitor and report the state of the environment at local and global levels, with a view to implementing actions for sustainable development (RAMOS *et al.*, 2014). In this perspective, indicators are important for monitoring environmental policy in different sectors of the economy (WURSTHORN *et al.*, 2011). Furthermore, eco-efficiency indicators contribute to comparing the evolution of eco-efficiency between countries, making it possible to establish goals and implement effective environmental policies (MOUTINHO *et al.*, 2017). Given that eco-efficiency is a management tool, its modeling and forecasting are relevant.

In this way, three possibilities are found in the literature. The former considers predicting the time series of inputs and outputs and then to measure the respective eco-efficiency (SADORSKY, 2021). Although viable, this approach may not be as promising. In particular, it is necessary to train models for each series of inputs and outputs. Another approach is to predict eco-efficiency based on covariates (SONG *et al.*, 2013; WANG; XIAO, 2017; LI *et al.*, 2017; LIU *et al.*, 2017; MOUTINHO *et al.*, 2017; MOUTINHO *et al.*, 2020; XIA *et al.*, 2021; MOUTINHO; MADALENO, 2021b; HE *et al.*, 2021). In this group, firstly, eco-efficiency is calculate by frontier methods (e.g. DEA, SFA, MPI). Next, different approaches and modeling are used (e.g. Tobit and logit models; quantile regression, fractional regression model, SVR) to measure the influence of several factors in eco-efficiency. In this case, the greatest difficulty is to obtain a vector of predictor variables to infer the next eco-efficiency values.

Lastly, one can resort to eco-efficiency time series forecasting via auto-regressive models (SONG *et al.*, 2013; LI *et al.*, 2017; CARBONI; RUSSU, 2018; CASTILHO *et al.*, 2021; CHEN *et al.*, 2022). In this cases, two types of approaches can also be delimited. There are models that consider exogenous variables, that is, eco-efficiency is predicted considering the lags of the eco-efficiency series, and a set of predictor variables (CASTILHO *et al.*, 2021). In this case, Castilho *et al.* (2021) applied Panel Autoregressive Distributed Lag (PARDL) to examine the impacts of the tourism sector on the general eco-efficiency of 22 countries in Latin America and the Caribbean from 1995 to 2016. Thus, after obtaining eco-efficiency, PARDL is used to evaluate the effects of eco-efficiency itself and other covariates. In the second case, the combination of past eco-efficiency values is used to predict contemporary values (SONG *et al.*, 2013; LI *et al.*, 2017; CARBONI; RUSSU, 2018; CHEN *et al.*, 2022). Illustrating these types of research, Song *et al.* (2013) considered GM and double Moving Average (MA) and combination of both for forecasting environmental efficiency and its influencing factors in China (2002-2010-2012). Carboni e Russu (2018) also applied GM for regional environmental and economic efficiency forecasting. However, this study considered 20 Italian regions from 2004 to 2011. On the other hand, Li *et al.* (2017) employed Radial basis function neural (RBFN) to predict regional energy efficiency in China. 30 regions in China were considered for a time horizon of 5 years. Finally, Chen *et al.* (2022) used the Panel vector autoregressive (PVAR) and Tobit panel models to analyze the effects on agricultural eco-efficiency of 31 provinces and cities in China from 2001 to 2020. Specifically, with PVAR for impulse-response functions of eco-efficiency and urbanization, that is, the effect of these variables over time.

Specifically, it is reiterated that the present study considers the latter. Contrariwise, it seems there are no studies for national eco-efficiency time series. Therefore, national eco-efficiency time series modeling and forecasting is of importance for economic segments, countries, policy makers, and society. Therefore, the present study makes use of DEA combined with WA, that is WDEA for eco-efficiency measuring (Section 2.3). In turn, details for eco-efficiency time series modeling and forecasting are presented in Section 2.4.

2.3 Window Data Envelopment Analysis

DEA is a nonparametric method to measure eco-efficiency of DMU using multiple inputs and outputs. DEA was proposed by Charnes *et al.* (1978), based on Farrell (1957). The objective of DEA is to build a frontier with the best practices of the DMUs, and calculate the relative efficiency of the other DMUs according to their distance from this frontier (WANG *et al.*, 2020). The eco-efficiency score varies between 0 and 1, and the same unit refers to the eco-efficient DEA, on the contrary, it is inefficient (KÜPELİ *et al.*, 2019; KINACI *et al.*, 2021). The seminal DEA model (CHARNES *et al.*, 1978) adopts Constant Returns to Scale (CRS), where the efficiency frontier is linear. The CRS model considers the proportionality between inputs and outputs. In general, the models can be input- or output-oriented. Input-oriented models obtain the optimal frontier by reducing inputs, keeping outputs unchanged. On the other hand, output-oriented models keep inputs constant by increasing outputs (COLUCCIA *et al.*, 2020). Data Envelopment Analysis with Constant Returns to Scale (DEA-CRS) input-oriented model in the multiplier form (dual problem) is described by Equation (2.2).

$$\begin{gathered}\max_{b_1,\ldots,b_m} \sum_{i=1}^{m} b_i v_{io}, \quad o = 1, \ldots, k \\ \text{subject to:} \begin{cases} \sum_{j=1}^{r} a_j u_{jo} = 1, \quad o = 1, \ldots, k \\ \sum_{i=1}^{m} b_i v_{io} - \sum_{j=1}^{r} a_j u_{jo} \leq 0 \end{cases}\end{gathered} \tag{2.2}$$

In this process, we have m outputs, where i is the index of the output and v_{io} corresponds to the value of the output i for the DMU o. Similarly, r inputs are adopted with j being the index of the input and u_{jo} representing the observed value of the input j of the DMU o. Thus, k optimization problems (which correspond to the number of DMUs) are solved simultaneously, obtaining the coefficients that weigh, respectively, the j-th input and the i-th output.

To analysis time series of DMUs eco-efficiency in DEA, two techniques are common. The first is the MPI. While the objective of DEA is to measure efficiency, MPI breaks down the productivity growth in relative efficiency and technological change, in two consecutive periods (FÄRE *et al.*, 1994). The second method is WA. WA was proposed by DEA creators (CHARNES *et al.*, 1983). The objective is to evaluate the performance of DMUs over time (ALKHARS *et al.*, 2022). DEA combined with WA is referred by WDEA. Therefore, WDEA is a method based on the moving average principle to measure eco-efficiency by treating each DMU in different periods as a separate DMU (SÁNCHEZ, 2018; PEYKANI *et al.*, 2021).

To apply window analysis it is necessary to split the time period, with size N, obtaining the window size, say n. Initially, following the moving average principle, the number of windows (w) can be obtained by Equation (2.3).

$$w = N - n + 1 \tag{2.3}$$

According to the literature, with k DMUs, the total number of distinct units of the WDEA analysis is equal to the product between k and w (COOPER *et al.*, 2006). In this context, the ideal window size (n) maximizes this product. Thus, using the first derivative with respect to n and equating it to zero for the constant k, we obtain n. The Equation (2.4) illustrates this process.

$$\frac{d}{dn}[k \cdot (N - 2n + 1)] = 0 \Rightarrow n = \frac{N+1}{2} \tag{2.4}$$

Although there are several studies applying this technique for computing n, there are criticisms. First, the classical rule obtains n as a function of the size of the time series (N). Thus, a larger N indicates a larger n. In turn, larger n makes difficult to obtain temporal changes in eco-efficiency (MIRMOZAFFARI *et al.*, 2020). On the other hand, the literature suggests the computation of DEA boundary with less loss of degrees of freedom (COOPER *et al.*, 2006). It can control the number of DMUs and variables (inputs and outputs). A common rule that relates these variables is the named Banker rule (GARFAMY, 2006; MA *et al.*, 2015). In WA, a alternative based in Banker rule may be suitable. Firstly, considering k DMUs, m outputs, and r inputs, Equation (2.5) expressed Banker rule.

$$k \geq max\{m \cdot r, 3 \cdot (m + r)\} \tag{2.5}$$

In WA, number of window (w) it has $k \cdot n$ distinct DMUs (MIRMOZAFFARI *et al.*, 2020). Considering that each window satisfies Banker rule, one has Equation (2.6).

$$k \cdot n \geq max\{m \cdot r, 3 \cdot (m + r)\} \tag{2.6}$$

Admitting that n belongs to the interval $[1, N]$, and that n must be minimum, this feature is justified by the fact that the smaller the n the lesser the consume of DMUs (MIRMOZAFFARI *et al.*, 2020). Gathering these properties, Equation (2.7) presents n for Banker rule in WDEA.

$$n = \min\left\{\max\left\{\frac{m \cdot r}{k}, \frac{3 \cdot (m + r)}{k}\right\}, N\right\}, n \in \mathbb{N}^* \tag{2.7}$$

In this study, the traditional and Banker rules are also considered as alternatives for computing n.

2.4 Time series analysis

In general, a time series is defined as a set of observations ordered sequentially in time (BOX *et al.*, 2015). Usually, the observations are collected in equally spaced time moments (WEI, 2006). In this way, Equation (2.8) presents the notation for a univariate time series y_t (COCHRANE, 1997).

$$\{y_1, y_2, \ldots, y_N\} \text{ or } \{y_t\}, t = 1, 2, \ldots, N \tag{2.8}$$

in which y_t represents the value of the time series at t and N the number of observations.

A feature of time series is that adjacent observations are dependent, which is called autocorrelation. So, time series analysis resorts to the concept of lags is interested in techniques for analyzing such autocorrelation (CHAN; CRYER, 2008; BOX *et al.*, 2015). In this research, the formalisms dedicated to time series are adopted to chronological eco-efficiency data modeling and forecasting. The modeling and forecasting include single (Section 2.4.1) and ensemble models (Section 2.4.2).

2.4.1 Single models

Single models are individual and direct functions of the data set under study. In this study, ARIMA, ETS, SVR, DTR, and LSTM are considered. The following subsections describe these formalisms.

2.4.1.1 Autoregressive integrated moving average

A Autoregressive (AR) process considers that the value of the time series at time t is a function of the past values of the time series (NIELSEN, 2019). Thus, AR model resorts to autocorrelated time series. The AR of order p, denoted by AR(p), is presented in Equation (2.9).

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t \tag{2.9}$$

in which y_t is the value of the time series at t; c is the constant of the regression; $y_{t-1}, y_{t-2}, \ldots, y_{t-p}$ are respectively the value of the time series at past times $t-1$, $t-2$, …, $t-p$; $\phi_1, \phi_2, \ldots, \phi_p$ are model coefficients; ε_t is a random error underlying the predictor when forecasting y_t.

The predicted value of y_t can also be influenced by residual errors from previous predictions (AGUSTIN, 2019). This process is called MA. So, a moving average model of order q, denoted by MA(q), is describe according to Equation (2.10)(NGUYEN, 2020).

$$y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} \tag{2.10}$$

in which y_t is the value of the time series at t; c is the constant of the regression; $\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-q}$ are the residuals of the predictor when predicting the past values of the series, $y_t, y_{t-1}, y_{t-2}, \ldots, y_{t-q}$, respectively; $\theta_1, \theta_2, \ldots, \theta_q$ are model coefficients, and ε_t is a random shock underlying the predictor when forecasting y_t.

Combining the models of Equations (2.9) and (2.10), we have the Autoregressive Moving Average (ARMA). Therefore, ARMA(p, q) process is presented in Equation (2.11).

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} \tag{2.11}$$

The ARMA model considers that the time series is stationary (BORUCKA *et al.*, 2021), i.e. statistical properties of the series, such as average and variance, do not depend on the time (HYNDMAN; ATHANASOPOULOS, 2018) . If the assumption is not verified, one may look for stationarity in the time series differentiation process (AGUSTIN, 2019). One procedure might be to differentiate the series. So, the first difference is $\Delta y_t = y_t - y_{t-1}$; the second difference is $\Delta(\Delta y_t) = \Delta^2 y_t = \Delta y_t - \Delta y_{t-1}$, and so on. For d differences, one has $\Delta^d y_t$. A time series that demands d differences for achieving a stationary behavior can be modeled by the ARIMA process. The ARIMA model is a popular statistical technique for time series forecasting using autocorrelation in data (US *et al.*, 2020; TUDOR; SOVA, 2021). Thus, considering that the time series was differentiated d times, Equation (2.12) presents the ARIMA model.

$$\Delta^d y_t = c + \phi_1 \Delta^d y_{t-1} + \phi_2 \Delta^d y_{t-2} + \ldots + \phi_p \Delta^d y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q}. \quad (2.12)$$

To choose the parameters p and q of the ARIMA model, the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) are widely applied (HOSSAIN *et al.*, 2012; NGUYEN, 2020; US *et al.*, 2020). However, to identify a parsimonious model, information criteria can be used. In the present project, Bayesian Information Criterion (BIC) is considered, following the literature (HOSSAIN *et al.*, 2012; NGUYEN, 2020). The BIC will be presented in Section 4.5.

2.4.1.2 Exponential smoothing

The ETS is a forecasting method based on the concept that the near the past observation to y_t, the more important the observation is. In this methods the forecasting assigns more weight to recent observations than to old values (HYNDMAN *et al.*, 2008). ETS is an acronyms for Error, Tendency, and Seasonality, the three components of the model. While ARIMA models describe the autocorrelations in the data, ETS models consider trend and seasonality (SADORSKY, 2021). In general, these exponential smoothing models include equations for forecasting the series based on its components (HYNDMAN; ATHANASOPOULOS, 2018) Thus, with regard to trend variation and seasonality in time series, several exponential smoothing forecast models are possible (LI *et al.*, 2018). The simplest ETS model is Simple Exponential Smoothing (SES). Holt's linear trend method, published in 1957, is an extension of SES (HOLT, 2004). This variation of the ETS considers the trend of the series. The Holt-Winters method is an extension of the Holt with trend for considering seasonality (WINTERS, 1960; HOLT, 2004). Further, one has two variations of these methods: additive and multiplicative error approaches.

Therefore, the ETS model identifies trend and seasonality and expresses this component by relationships (additive and multiplicative) utilizing exponential smoothing (MA; KIM, 2015). The relationship between components produces several ETS models. Thus, ignoring the error component and following (HYNDMAN *et al.*, 2008), Table 1 summarizes fifteen exponential smoothing methods.

Table 1 – Types of exponential smoothing models according to trend and seasonality components

Trend component	Seasonal component		
	N(None)	A(Additive)	M(Multiplicative)
N(None)	N, N	N, A	N, M
A(Additive)	A, N	A, A	A, M
A_d(Additive damped)	A_d, N	A_d, A	A_d, M
M(Multiplicative)	M, N	M, A	M, M
M_d(Multiplicative damped)	M_d, N	M_d, A	M_d, M

In general, these presented methods can be written from a state space model (HYNDMAN; KHANDAKAR, 2008). Thus, the Equation (2.13) describes the relationship between unobserved state $\ddot{x}_t$ and y_t, while Equation (2.14) is known as the transition (or state) equation (HYNDMAN *et al.*, 2008).

$$y_t = \delta \ddot{x}_{t-1} + \epsilon_t \tag{2.13}$$

$$\ddot{x}_t = F \ddot{x}_{t-1} + g \epsilon_t \tag{2.14}$$

in which y_t is the value of the time series at t; $\ddot{x}_t$ is a space vector given by $\ddot{x}_t = (\ell_t, b_t, s_t)$; δ, F, and g are coefficients (HYNDMAN; KHANDAKAR, 2008).

Given the variety of models, the optimal ETS is obtained by minimizing an information criterion or a prediction error measure (BORUCKA *et al.*, 2021). In the same way as the ARIMA model, BIC is used in the present work (see Section 4.5).

2.4.1.3 Support vector regression

Support Vector Machines (SVM) are supervised learning algorithms developed initially by Vapnik (1999) and useful for both classification and regression problems (POITIER; CHO, 2011). The objective of this algorithm is to maximize the margin around the hyperplane that separates data points (NILASHI *et al.*, 2017). In case of regression, the term SVR is considered. Given a training data set $\{(x_t, y_t)\}_{t=1}^{N}$, in which $x_t \in \mathbb{R}^I$, is a I-dimensional input vector and $y_t \in \mathbb{R}$ is a scalar output (LEE *et al.*, 2020), the main objective is to search the function $f(x)$ with at most one deviation ϵ to y_t. Equation (2.15) denotes $f(x)$:

$$f(x) = \langle \eta, x \rangle + b \tag{2.15}$$

in which b is the model intercept; η is a weight vector;

The task involves determining the weights and intercept. To do this, a constrained optimization problem can be used (Equation 2.16).

$$\begin{gathered}\min_{\eta, b, \xi_t^+, \xi_t^-} \frac{1}{2}\|\eta\|^2 + C\sum_{t=1}^{N}(\xi_t^+ - \xi_t^-) \\ \text{subject to:} \begin{cases} y_t - f(x_t) \leq \epsilon + \xi_t^+ \\ f(x_t) - y_t \leq \epsilon + \xi_t^- \\ \xi_t^+, \xi_t^- \geq 0 \end{cases}\end{gathered} \tag{2.16}$$

in which: C is the regularization constant for the number of errors in the training set; ξ_t^+ and ξ_t^- are slack variables with respect ϵ; ϵ is the error tolerance.

An important characteristic of SVM and SVR is the named kernel function, say $K(.)$. Particularly, two established kernel functions are the Radial Basis function (2.17) and the Sigmoid function (2.18) (GÉRON, 2022).

$$K(x_t, x) = \exp\left(-\gamma\|x_t - x\|\right) \tag{2.17}$$

$$K(x_t, x) = \tanh(\gamma(x_t \cdot x) + d) \tag{2.18}$$

in which γ is a parameter defined in the kernel function.

Therefore, introducing the kernel function $K(x_t, x)$ and applying the Lagrange Multiplier to this optimization problem, Equation (2.15) is described by Equation (2.19).

$$f(x) = \sum_{t=1}^{N}(\alpha_t - \alpha_t^*)K(x_t, x) + b \tag{2.19}$$

in which α and α^* are Lagrange multipliers.

In this study, SVR is used for time series forecasting. Thus, this method uses the autocorrelation of the time series. Therefore, the input values are the past observations of the time series (LEE *et al.*, 2020). Considering the lag p, we can write the set of inputs $x_t = (y_{t-1}, \ldots, y_{t-p})$. SVR is therefore a AR model with order p.

For fitting SVR it is necessary to optimize the parameters of the model. In this case, the technique used is called hyperparameter optimization. The hyperparameters of SVR are C, ϵ and γ (from kernel function), considering radial basis and sigmoid kernels. Section 4.3 presents the methods used to optimize hyperparameters in SVR.

2.4.1.4 Decision tree regression

Decision Tree (DT) is a supervised machine learning algorithm that can be used for classification and regression tasks (D'AMATO *et al.*, 2022; GÉRON, 2022). Particularly, in this study, the focus is on regression (called DTR). The main difference is that the target variable is not categorical, but numerical (SPILIOTIS *et al.*, 2022). The deeper the tree, the more complex the decision rules and the more adjusted the model will be (BERNARDO *et al.*, 2023). Thus, according to the literature (SUTTON, 2005; SPILIOTIS *et al.*, 2022), in DTR, each terminal node is assigned a predicted value.

For training trees, Classification And Regression Trees (CART) algorithm, developed by Breiman *et al.* (1984) is used. According to the literature (DECONINCK *et al.*, 2005; GÉRON, 2022), CART follows some steps for adjusting DTR. First, divide the training set into two subsets using a characteristic ψ and a threshold t_ψ. Next, it divides the subsets using the same logic, and again, recursively. The division can stop when the maximum depth of the tree is reached, for instance. In this process, the algorithm minimizes the mean squared error. Thus, Equation (2.20) presents CART cost function for DTR (GÉRON, 2022).

$$J(\psi, t_\psi) = \frac{\mu_{left}}{\mu} MSE_{left} + \frac{\mu_{right}}{\mu} MSE_{right} \tag{2.20}$$

in which μ_{node} is the number of instances in a node μ; μ_{left} and μ_{right} are the child nodes resulting from the split; MSE denotes mean squared error ($MSE_{node} = \frac{1}{\mu_{node}} \sum_{i \in node} (y_i - \hat{y}_{node})^2$, in which $\hat{y}_{node}$ is the mean of the target variable in node μ and y_i is the $i - th$ instance of the target variable.

DTR are also prone to overfitting (GÉRON, 2022). For this reason, it is necessary to adjust the model hyperparameters. The model hyperparameters are important for its performance, so correctly choosing values increase its performance (ALHAKEEM *et al.*, 2022). In DTR, the Grid search cross-validation (GridSearchCV) technique can be applied. This method and the hyperparameters to optimize DTR are described in Section 4.3.

2.4.1.5 Artificial neural network

Artificial neural networks (ANN) is a computational framework used for modeling several problems (ANOUZE; BOU-HAMAD, 2019; SALES, 2019). Thus, ANN simulate characteristics of the human nervous system (AMIRI; VENTELOU, 2011; GÉRON, 2022). Neural networks consist of nodes (neurons) organized into layers, with each node being connected to the nodes in the preceding layer through a network of weighted connections (ANOUZE; BOU-HAMAD, 2019). Several ANNs are documented in the literature for time series forecasting (MUSHTAQ *et al.*, 2019). In this study, LSTM is used.

LSTM is a special type of recurrent neural network that learns long-term dependence (SUN *et al.*, 2021; MA; WANG, 2022). Proposed by Hochreiter e Schmidhuber (1997), LSTM addresses gradient vanishing problems in predicting samples with long sequences. According to Géron (2022), LSTM will perform better; training will converge faster and will be able to detect long-term dependencies in the data. In practical terms, LSTM can preserve or discard information from its memory. This is due to the LSTM memory cell which consists of a neuron with internal recurrence and three gates: forget, input and output gate (SUN *et al.*, 2021; GÉRON, 2022; ESPARZA-GÓMEZ *et al.*, 2023).

The forget gate is responsible for filtering information, whether it is removed or preserved (SUN *et al.*, 2021; ESPARZA-GÓMEZ *et al.*, 2023). The Equation (2.21) presents the forget gate.

$$f_t = \sigma(W_{if}x_t + b_{if} + W_{hf}h_{t-1} + b_{hf}) \tag{2.21}$$

in which σ is a sigmoid activation function; x_t is a input vector; $h(t)$ is the hidden layer vector; W and b are the weights and biases adjusted in the training stage, respectively.

The input gate controls what new information can be entered into the network (GÉRON, 2022; ESPARZA-GÓMEZ *et al.*, 2023). Thus, the set of Equations (2.22) and (2.23) described the input gate. Particularly, Equation (2.23) is a vector with new candidates that will be added to the neural network memory.

$$i_t = \sigma(W_{ii}x_t + b_{ii} + W_{hi}h_{t-1} + b_{hi}) \tag{2.22}$$

$$\tilde{C}_t = \tanh(W_{ic}x_t + b_{ic} + W_{hc}h_{t-1} + b_{hc}) \tag{2.23}$$

in which *tanh* is the hyperbolic tangent activation function. Particularly, *tanh* function is useful because it maps the inputs and transforms them into outputs in the range between -1 and 1. Mathematically, *tanh* is defined by $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, in which e is the exponential constant ($e \approx 2.7182$).

The previous state of the cell (C_{t-1}) is updated to the new state (C_t) by applying Equation (2.24) (ESPARZA-GÓMEZ *et al.*, 2023).

$$C_t = f_t \cdot C_{t-1} + i_t \cdot \tilde{C}_t \tag{2.24}$$

Lastly, the output gate decides whether information from the current memory cell contributes to the LSTM cell output activation compute (SUN *et al.*, 2021; MA; WANG, 2022). The sigmoid and hyperbolic tangent functions are used. The output gate is shown in the Equation (2.25).

$$o_t = \sigma(W_{io}x_t + b_{io} + W_{ho}h_{t-1} + b_{ho}) \cdot \tanh(C_t) \tag{2.25}$$

All three gate controllers use logistic sigmoid activation function and produce outputs between from 0 to 1 (SUN *et al.*, 2021; GÉRON, 2022). When it is 0, the gate closes and if 1, the gate opens. Furthermore, to training LSTM, gradient descent algorithm such as the back propagation through time can be utilized (WERBOS, 1988; SUN *et al.*, 2021).

2.4.2 Ensemble learning

Ensemble learning (or just ensemble) is a technique for aggregating single predictors. Predictive model is obtained from weak models using combination, averaging and voting approach (GÉRON, 2022). Additionally, the ensemble technique can reduce the concern of overfitting (ULLAH *et al.*, 2022). Particularly, literature highlights three ensemble learning algorithms: bagging, boosting, and stacking (RIBEIRO; COELHO, 2020; REN *et al.*, 2021; GÉRON, 2022).

In bagging and stacking, the objective is to train models to aggregate predictions (WOLPERT, 1992; GÉRON, 2022). Further, in general, the same algorithm is used for each predictor, but trained on different random subsets of the training set (GÉRON, 2022). In other words, it is characterized by creating multiple sampling using the bootstrap technique refitting for the same set of data by constructing multiple prediction and generating an aggregate prediction (RIBEIRO; COELHO, 2020). General idea of boosting is to train the models by sequencing, each one correcting the previous one (GÉRON, 2022). In this study, therefore, ensemble methods are used. Particularly, SA, SM and MV (FIRMINO *et al.*, 2014; SALES, 2019); RFR (BREIMAN, 2001) and XGB (FRIEDMAN, 2001) are applied.

SA and SM are non-parametric statistics, i.e. the arithmetic mean and median of forecasts, respectively. Specifically, the arithmetic mean is the sum of the values in the set divided by the total number of observations . In the case of this study, the set mentioned is the predictions of the models. Thus, the median is the central value for a set of ordered data (KAUR *et al.*, 2018). On the other hand, the prediction by MV is calculated considering the weights of the error covariance matrix in single models (FIRMINO *et al.*, 2014). In particular, the variance and covariance of the errors of each model in relation to the other single models are involved (SALES, 2019).

RFR algorithm is an ensemble introduced by Breiman (2001). Since the individual decision tree can overfit the data, the RFR algorithm is a tool that can predict effectively without overfitting (REN *et al.*, 2021). In general, RFR is trained using the bagging method (GÉRON, 2022). Thus, the RFR algorithm is a combination of decision trees that depends on the values of the random vector taken from an independent sample and with equal distribution for all trees in the forest (BREIMAN, 2001). Furthermore, RFR introduce an extra randomness in the tree training. In this sense, instead of looking for the best feature when splitting the node, these algorithm search a random subset of these characteristics enabling a greater diversity of trees (GÉRON, 2022). Particularly, XGB is a boosting type ensemble. Specifically, this algorithm combines gradient descent and boosting (ESPARZA-GÓMEZ *et al.*, 2023). The concept of XGB was introduced by Friedman (2001) in the 1990s. XGB is an algorithm that sequentially adds predictions together by adjusting a new predictor from residual errors obtained by the past predictor (GÉRON, 2022).

3 PROPOSED METHOD

This chapter is dedicated to the proposed method. Initially, the method for calculating the window size (n) in WDEA is presented. Then, an approach is described for studying small eco-efficiency time series.

According to Section 2.3, some methods for calculating window size in WDEA are documented. However, traditional and Banker rules do not consider eco-efficiency behavior when calculating n. The literature has advanced in this discussion. Particularly, in the method proposed by Hao *et al.* (2013) the optimal window size is determined by the difference between the average eco-efficiency of each window and the mean period of the study. Several studies has employed this method (LIN *et al.*, 2018; KYRGIAKOS *et al.*, 2021; PISHGAR-KOMLEH *et al.*, 2021). Although the window size based on average eco-efficiency produces good analyses, alternatives considering the distribution of eco-efficiency among DMUs are promising.

When studying the dynamic evolution of eco-efficiency between territories, the main idea is the convergence (and divergence) analysis (KOUNETAS *et al.*, 2021). Particularly, convergence analysis is an economic theory that refers to the process in which economies approach each other in terms of some specific indicators. In essence, backward countries learn from the past of developed countries, allowing them to increase environmental and economic efficiency (SUN *et al.*, 2020). Classical literature mainly presented two types of convergence: σ-convergence and β-convergence (BAUMOL, 1986; BARRO; MARTIN, 1990; MARTIN, 1996). Convergence clubs are also documented in the literature (CAMARERO *et al.*, 2014), although they are not the focus of this work. The simple analysis is σ-convergence, employing with frequency in country eco-efficiency (NEŽINSKỲ, 2014). In this case, the basic idea is to verify the dispersion of score eco-efficiency between DMUs over time (CAMARERO *et al.*, 2014). It can be said that σ-convergence involves a cross-sectional eco-efficiency dispersion and implies reducing this dispersion in time (SUN *et al.*, 2020). Therefore, eco-efficiency behavior affects convergence and divergence. Thus, the greater the presence of territories on the optimal frontier, that is, with eco-efficiency equal to 1, the greater the dispersion, implying divergence (NEŽINSKÝ, 2014; LIN *et al.*, 2018). However, in window analysis applications, when the window size increases, the eco-efficiencies should be averaged (SANTANA *et al.*, 2014; CAMIOTO *et al.*, 2016). As a result, in years in which the territories perform well, they end up being leveled at the average of other years. From this perspective, it is of interest to obtain a method that determines windows in which benchmarks can be studied. Thus, the window size must satisfy the greatest dispersion between eco-efficiencies in order to highlight the benchmarks from inefficient countries. In this way, the window size is suggested based on the eco-efficiency divergence.

Therefore, the motivation for studying maximum dispersion in eco-efficiency considers the dissimilarity between territories in terms of eco-efficiency. Thus, the optimal window size is obtained from the window that maximizes the dispersion of eco-efficiency. Maximizing dispersion therefore implies increasing eco-efficiency divergence. So, the objective is to research n using dispersion metrics on the eco-efficiency of territories. Most studies that evaluate eco-efficiency convergence and divergence use standard deviation and coefficient of variation (NEŽINSKÝ, 2014; XU *et al.*, 2020; KOUNETAS *et al.*, 2021; ZHANG *et al.*, 2022). In this work, the coefficient of variation is not used. Coefficient of variation is influenced by eco-efficiency average. Thus, the average eco-efficiency decreases when the sample of DMUs increases (ZHANG; BARTELS, 1998; HUANG; ELING, 2013), thus affecting the coefficient of variation, implying the same behavior of this metric in the window size n. Therefore, in addition to the standard deviation, other metrics defined below are used. In this study, in general, the function $s(n)$ denotes the dispersion metric s for the window size n. Therefore, the optimal n satisfies the maximum of the function $s(\cdot)$, considering $1 \leq n \leq N$. This condition is denoted in Equation (3.1).

$$\max_{n \in \{1,\ldots,N\}} s(n) \tag{3.1}$$

For optimal window size, $s(n)$ can denote any dispersion function (e.g. Standard Deviation (SD) and Mean-Absolute Deviation (MAD)). In Chapter 4, Section 4.5, dispersion metrics are discussed.

With n it is possible to obtain the eco-efficiency time series using WDEA. If the window size is greater than 1, the technique for aggregating eco-efficiency in the window is necessary, e.g. average (SANTANA *et al.*, 2014; CAMIOTO *et al.*, 2016). Considering $1 \leq n \leq N$, the length of the eco-efficiency time series decreases as n increases. In the special case for $n = N$ (case the window size is equal to the time series size), this eco-efficiency time series in a single window is just one point.

Thus, a recurring issue in eco-efficiency time series is their limited size. Particularly, these are short series (sometimes very small). To increase the number of individual observations, this study pooled the time series. The idea is to group the series from different countries into a single series. For each time series, the effect of its country is considered. Given that they are autoregressive models, for each time series the past values are considered, taking p lags. Therefore, the structure defined in the function expressed in Equation (3.2) is adopted.

$$\mathbf{Y} = f(\mathbf{X}, \mathbf{D}) \tag{3.2}$$

The model presented in Equation 3.2 can be written in matrix structure. Therefore, Equation 3.3 shows the matrices for the components of the suggested structure.

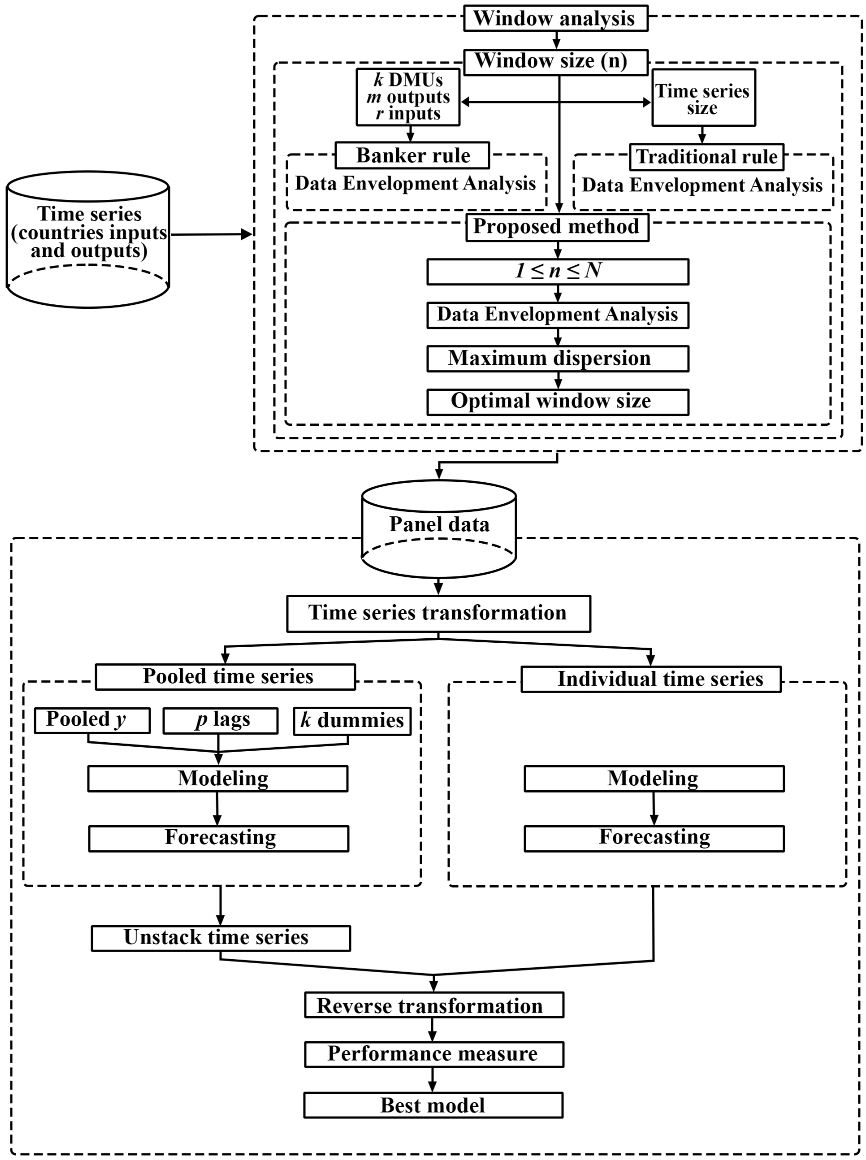
$$\mathbf{Y} = \begin{bmatrix} y_{1,1} \\ y_{2,2} \\ \vdots \\ y_{k,t} \\ \vdots \\ y_{1,1} \\ y_{2,2} \\ \vdots \\ y_{k,t} \end{bmatrix}, \mathbf{X} = \begin{bmatrix} y_{1,t-1} & \cdots & y_{1,t-p} \\ y_{2,t-1} & \cdots & y_{2,t-p} \\ \vdots & \ddots & \vdots \\ y_{k,t-1} & \cdots & y_{k,t-p} \\ \vdots & \ddots & \vdots \\ y_{1,t-1} & \cdots & y_{1,t-p} \\ y_{2,t-1} & \cdots & y_{2,t-p} \\ \vdots & \ddots & \vdots \\ y_{k,t-1} & \cdots & y_{k,t-p} \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 1 & \dots & 0 \\ 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \\ 0 & \dots & 1 \\ 0 & \ddots & \vdots \\ 0 & \dots & 1 \end{bmatrix} \tag{3.3}$$

in which $\mathbf{Y}$ is a vector of pooled time series; $\mathbf{X}$ is a matrix with p lags for the series; $\mathbf{D}$ is a matrix of dummies variables for the countries.

Therefore, following this structure, each variable has $N \cdot k$ observations, where N is the size of the individual time series and k is the number of countries. This structure is similar to panel data approaches (WOOLDRIDGE, 2010; HSIAO, 2022). In particular, Hsiao (2022) highlights that the structure can increase the sample size from data points, as well as expand the degrees of freedom, reducing collinearity between variables. In this study, therefore, all these characteristics are present in the pooled structure. Although the literature uses panel data to analyze various factors that affect eco-efficiency (BELUCIO; GUARINI, 2023), this study focuses on predicting eco-efficiency time series based on the autocorrelation of the series. Therefore, this framework is used to train and test models for national eco-efficiency prediction. Then, the predicted values are unstack for each country.

This procedure applied for single models (SVR, LSTM and DTR) and ensemble learning (SA, SM, MV, RFR and XGB). Symmetrically, these models will also be fitted for the individual series. In this group, ARIMA and ETS are included. The Figure 2 describe the methodological procedure employing in this study.

Figure 2 – Framework of the proposed method



4 METHODOLOGICAL PROCEDURE

This chapter is concerned with the methodology of this work. The methodological procedure represents the path for development of research. The study is divided in the following contents: data set (Section 4.1); data preprocessing (Section 4.2); modeling (Section 4.3); forecasting (Section 4.4); evaluation of models by performance measures (Section 4.5).

4.1 Data

For building WDEA, the data set must be similar to a production process with inputs and outputs. Following economic theory, production involves several ingredients (e.g. capital stock, land, labor force, and energy consumption). The aggregate production is measured by GDP (DEMIRAL; SAĞLAM, 2021). In the production process, undesirable outputs are generated in economic activities (e.g. water pollution, smoke and carbon dioxide (CO_2) emissions) (KOOPMANS, 1951). In this research, capital stock, labor force and arable area are factors of production to generate GDP in countries. These three variables and the energy consumption are inputs. GDP is the desirable (or good) output of the model. The undesirable (or bad) output is greenhouse gas emissions. The choice of these variables is based on the literature (ROBAINA-ALVES *et al.*, 2015; SADORSKY, 2021; MOUTINHO; MADALENO, 2021b). Table 2 presents the variables selected to measure the national eco-efficiency. The variables are obtained from the World Bank (World Bank, 2024) and Our World Data (RITCHIE *et al.*, 2023).

Table 2 – Economic and environmental variables for measuring national eco-efficiency

Type	Variable	Definition	Unit
Input	Arable land	Includes land under temporary crops, temporary meadows for mowing or for pasture, land under market or kitchen gardens, and land temporarily fallow. Land abandoned as a result of shifting cultivation is excluded.	% of land area
	Labor force	Corresponding people ages 15 and older who work in the production of goods and services in a specific period.	% of total population
	Gross fixed capital formation	Includes land improvements and acquisition of factories, machinery and equipment. It also includes the construction of roads and railways, as well as schools, hospitals, residential and commercial and industrial buildings.	% of GDP
	Primary energy	Refers to primary energy. Primary energy is the energy as it is available as resources. This relates to the coal before it has been burned; the uranium; or the barrels of oil.	joules of energy
Desirable output	Gross domestic product (GDP)	GDP is the sum of the gross value of production added by all resident producers in the economy plus taxes on production and minus subsidies not included in the value of products.	Current US
Undesirable output	Total greenhouse gas emissions	It comprises total CO2 excluding short-cycle biomass burning (e.g. agricultural and savannah residues) and including other biomass burning (e.g. forest fires). In addition, it includes all anthropogenic sources of CH4, sources of N2O and fluorinated gases (HFCs, PFCs and SF6).	kt of CO2 equivalent

Source: Ritchie *et al.* (2023) and World Bank (2024).

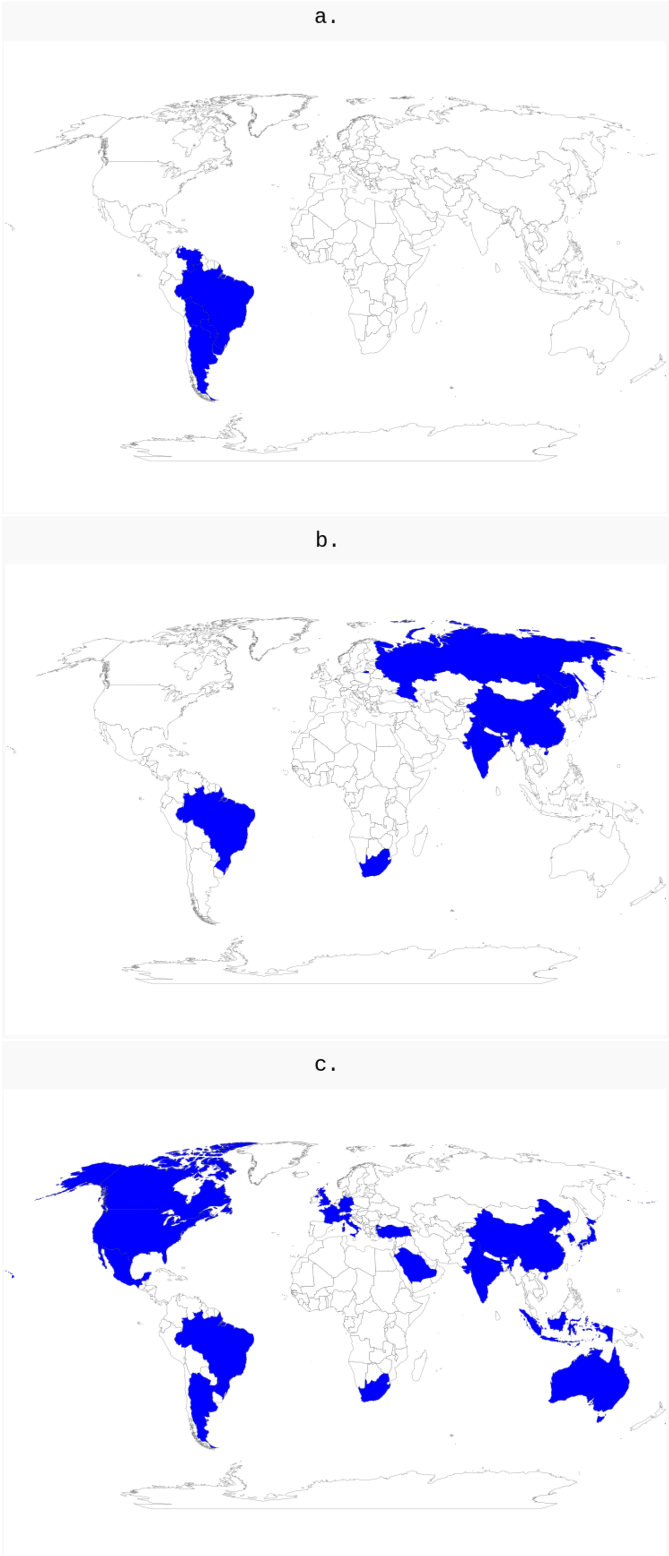
In general, in production processes, outputs are maximized. However, these undesirable outputs constitute a global concern because they affect the climate

(ZADMIRZAEI *et al.*, 2023). Therefore, unlike GDP, it must be minimized. In oriented models, several techniques are adopted to model undesirable outputs (SCHEEL, 2001). In this study, the output of the WDEA model is the ratio of good to bad output. Therefore, the ratio between GDP and total greenhouse gas emissions is considered (see Table 2). This procedure is inspired by the literature (ROBAINA-ALVES *et al.*, 2015).

Thus, for countries, multiple groups can be used for time series eco-efficiency analysis. In this research, three case studies on national eco-efficiency are considered. The first case study considers Mercosul, which includes Argentina (ARG), Brazil (BRA), Paraguay (PRY), Uruguay (URY), Venezuela (VEN) and Bolivia (BOL). Officially, Bolivia joined Mercosul in December 2023. However, Venezuela is suspended from the group, which is why it is not considered in this study. Particularly, Mercosul is a customs union founded in 1991 following the Treaty of Asunción. The objective is to create a common market for the free movement of goods, services, capital, and labor (VIEIRA *et al.*, 2014). The second case considers the BRICS. Specifically, BRICS originated in 2006, and are emerging economies (SUN; HUANG, 2021). BRICS is composed by Brazil (BRA), China (CHN), India (IND), Russian Federation (RUS), and South Africa (ZAF)[1]. Finally, the eco-efficiency of G18 countries is studied. In this perspective, 18 economies make up the G18: Argentina (ARG), Australia (AUS), Brazil (BRA), Canada (CAN), China (CHN), Germany (DEU), France (FRA), United Kingdom (GBR), Indonesia (IDN), India (IND), Italy (ITA), Japan (JPN), South Korea (KOR), Mexico (MEX), Saudi Arabia (SAU), Turkey (TUR), United States (USA), and South Africa (ZAF). Figure 3 represents the countries considered in each case. Further, in all cases the eco-efficiency time series considers the period from 1995 to 2020.

[1] Although Egypt, Ethiopia, Iran and the United Arab Emirates joined the group in January 2024, this study only considers five BRICS countries.

Figure 3 – Countries considered in each case: a. Mercosul, b. BRICS, c. G18



4.2 Data preprocessing

Data pre-processing consists of steps to process the data set that include from obtaining variables to transforming and pooling series. The first part consists of obtaining the eco-efficiency time series. Then, with the set of economic and environmental variables (inputs and outputs), WDEA is applied. In this study, to optimize models, maps, graphs and tables, the `Python` language is used. Specifically, `Python` 3 was used and the runtime environment was a `Colab` notebook. Thus, to optimize WDEA, `DEA` function of the `pyStoNED` package is used. In this case, window size, n, is computed via proposed method (Chapter 3). Returning to the concepts presented in the Chapter 3, an eco-efficiency matrix is obtained considering $1 \leq n \leq N$. Window size optimal n is obtained. Traditional and Banker rules are also be considered as an illustration. Dispersion metrics are used in window size optimization (Section 4.5). The result is the set of countries eco-efficiency time series. Furthermore, the data set is split: 75% for training data set and 25% for test data set. Countries with eco-efficiency equal to 1 (reference units) in more than 50% of the years in the time series of the training data set are removed. Then, the series of the training data set were standardized. Thus, the Equation 4.1 applies:

$$y_t^{'} = \frac{y_t - \overline{y}}{sd_y} \tag{4.1}$$

where $y_t^{'}$ is the standardized time series value y_t; $\overline{y}$ is the average of the time series y; sd_y is the standard deviation of y.

To build the structure with the pooled time series with lags and dummies for the countries, three steps are used: (i) a single variable for the series is obtained with pooled individual series; (ii) for each time series is obtained the lag (p); (iii) for each individual time series, countries are also pooled. To capture the effects of countries on eco-efficiency, a categorical feature encoder is used to obtain a matrix of binary variables (dummies) for the set of countries. For this transformation, the

`OneHotEncoder` function from the `sklearn` library is used. The final data structure contains $p+k+1$ columns, composed of the pooled time series, p lags, k dummies, in which k is the number of countries.

4.3 Modeling

The process of modeling involves constructing time series models. The training data set is used for models fitting. ARIMA and ETS are automatically adjusted. In both cases, the best model minimizes BIC. Particularly, to fit ARIMA `pmdarima.arima` (SMITH *et al.*, 2017) is used. The boolean `false` is assigned to the argument `stepwise`. In turn, the objective is deep search of model parameters (HYNDMAN; KHANDAKAR, 2008). Further, ETS is automatically adjusted by the `sktime` function (LöNING *et al.*, 2024).

For individual time series and pooled approaches, SVR, DTR, RFR and XGB are optimized using the `sklearn` (PEDREGOSA *et al.*, 2011) library. Hyperparameter optimization is used in these machine learning models. Thus, several methods are present in the literature for hyperparameter optimization in machine learning algorithms (e.g. genetic algorithms and particle swarm optimization) (APRIYADI *et al.*, 2023). Two techniques have stood out: Randomized search (RandomizedSearch) and Grid search (GridSearch). RandomizedSearch performs a random search in the hyperparameter space, while GridSearch searches in a predefined grid of hyperparameters (GÉRON, 2022; YENNIMAR *et al.*, 2023). For improving the precision of the forecast, cross-validation (CV) method is employed with GridSearch, named GridSearchCV. From this perspective, CV randomly divides the training set into k (k-folds) different subsets and model adjusted and evaluated on such sets. Further, it seeks the best combination of hyperparameters that provide optimal results for the model performance (ADNAN *et al.*, 2022).

Thus, GridSearchCV adjust the hyperparameters considering a range of specified parameters, train the estimator with the adjusted hyperparameters and find those with greater accuracy for the model (ZOU *et al.*, 2022). Therefore, all machine learning models require hyperparameter tuning.

For SVR, the possibilities for C, ϵ, γ and kernel functions are defined and searched. In DTR and RFR, is defined the maximum depth of the tree (`max_depth`), the minimum number of samples required to split an internal node (`min_samples_split`) and the number of features to consider when searching for the best split (`max_features`). Further, the RFR also requires the number of trees in the forest (`n_estimators`). Finally, to optimize XGB, in addition to `max_depth` and `n_estimators`, it is necessary to define learning rate shrinks of each tree (`learning_rate`) and the fraction of samples to be used for fitting the individual base learners (`subsample`) (HASTIE *et al.*, 2009). In Table 3, the list of hyperparameters is described.

Table 3 – Lists of values for hyperparameters to be optimized

Type	Hyperparameter	SVR	DTR	XGB	RFR
Individual	C	[11, 15, 19]	N/A	N/A	N/A
	ϵ	[0.1, 0.5, 1]	N/A	N/A	N/A
	`kernel`	[`rbf`, `sigmoid`, `poly`]	N/A	N/A	N/A
	γ	[0.05, 0.1, 0.25]	N/A	N/A	N/A
	`max_depth`	N/A	[2, 5, 9]	[3, 11, 18]	[3, 11, 18]
	`min_samples_split`	N/A	[8, 9, 12]	N/A	[5, 12, 23]
	`max_features`	N/A	[11, 13, 17]	N/A	[7, 9, 13]
	`n_estimators`	N/A	N/A	[45, 150, 350]	[25, 55, 85]
	`learning_rate`	N/A	N/A	[0.001, 0.005, 0.01]	N/A
	`subsample`	N/A	N/A	[0.1, 0.5, 0.75]	N/A
Pooled	C	[1, 5, 100]	N/A	N/A	N/A
	ϵ	[0.001, 0.01, 0.05]	N/A	N/A	N/A
	`kernel`	[`rbf`, `sigmoid`, `poly`]	N/A	N/A	N/A
	γ	[0.01, 0.05, 0.1]	N/A	N/A	N/A
	`max_depth`	N/A	[5, 7, 11]	[7, 14, 28]	[7, 14, 28]
	`min_samples_split`	N/A	[9, 13, 19]	N/A	[11, 16, 55]
	`max_features`	N/A	[9, 11, 15]	N/A	[7, 13, 25]
	`n_estimators`	N/A	N/A	[150, 350, 550]	[180, 300, 450]
	`learning_rate`	N/A	N/A	[0.0001, 0.001, 0.005]	N/A
	`subsample`	N/A	N/A	[0.25, 0.5, 0.75]	N/A

Note: `N/A` indicates that the algorithm does not have the parameter or has not been optimized.

Therefore, this study considers GridSearchCV for hyperparameter tuning in machine learning models. For hyperparameters tuning using GridSearchCV, `sklearn` library is used. Furthermore, in CV, for k-fold, $k = 6$ is used in both the individual and pooled approaches.

Furthermore, for LSTM training, the `keras` (CHOLLET *et al.*, 2015) and `tensorflow` (ABADI *et al.*, 2015) packages were used. The LSTM for individual and pooled series were trained with 150 and 450 epochs, respectively. In both cases, the optimizer is `adam` and the loss function is `mean squared error`. For training SVR, DTR, RFR, XGB, and LSTM lags of the time series are used as inputs and the series is the output. In the pooled approach, in addition to lags, the individual effects of territories are considered. To train the machine learning models, relevant lags are defined. For this purpose, p models are adjusted, where p is the lag. The first model contains lag one, the second lag two, and so on, until the p-th model with p lag. In each model, the error metric Root Mean Squared Error (RMSE) is calculated. In the list of models, the model with the lowest RMSE is selected. The maximum lag adopted was $p = 6$. Combinations by SA, SM and MV were performed using all single models (except RFR). That is, RFR, as an ensemble based on combinations of single models, is not used to generate predictor combinations. On the other hand, XGB, although ensemble, uses the sequential errors of individual models. In the modeling step, weights of MV are calculated by residuals of each model included in the ensemble. These weights of MV are used for forecasting steps considering the predictions of single models and XGB as inputs. Combinations by SA and SM do not need modeling steps. Forecasts are calculated by directly involving forecasts of single models and XGB. Thus, `mean` and `median` functions are applied for SA and SM, respectively.

4.4 Forecasting

The next step is to predict the time series from the models, considering the training set. In all cases, the prediction is carried out one-step-ahead. Further, interactive prediction is adopted. This type of forecast is based on Hyndman e Athanasopoulos (2018). For one-step-ahead prediction, the model uses past time series values. In the next forecast, the previously predicted value and series values are used as input to the model. For individual time series, the forecasting is realized in each time series. On the other hand, for the pooled structure, predictions are made considering the pooled and unstacked for each time series. In both types, the reverse transformation is applied, that is, the reverse of the Equation 4.1 is used. Finally, RMSE is calculated for the forecasts.

4.5 Performance measures

For selection of specific models (ARIMA and ETS), this study uses information criteria. Particularly, the BIC is employed. The BIC was developed by Schwarz (1978). BIC is presented in Equation (4.2):

$$BIC = -2\ln(\mathcal{L}) + \phi \ln(N) \tag{4.2}$$

in which $\mathcal{L}$ is the maximum value of the likelihood function for the analyzed model in the face of the residuals of the adjusted model; ϕ represents the number of model parameters; and N the sample size.

According to Chapter 3, to choose the ideal window size (n), dispersion metrics are used. Thus, $s(n)$ denotes any dispersion metric for window size n. Particularly, this study considering SD and MAD. Then, Equations (4.3) and (4.4) present SD and MAD, respectively.

$$SD(n) = \sqrt{\frac{1}{\Omega}\sum_{i=1}^{\Omega}(y_i - \overline{y})^2} \tag{4.3}$$

$$MAD(n) = \frac{1}{\Omega}\sum_{i=1}^{\Omega}|y_i - \overline{y}| \tag{4.4}$$

in which y_i is the eco-efficiency score of the DMU i; and $\overline{y}$ is the average of the ecoefficiencies $(y_1, y_2, \ldots, y_\Omega)$, $\overline{y} = \frac{1}{\Omega}\sum_{i=1}^{\Omega}\theta_i$; Ω is the number of distinct DMUs in the window size equal to n ($\Omega = k \cdot n$).

After estimating the time series models, it is necessary to evaluate the performance in terms of predictive quality. The literature uses measures based on errors (WANG; XIAO, 2017; NGUYEN, 2020; WANG *et al.*, 2021; JAUHAR *et al.*, 2022). Specifically, RMSE is adopted. Thus, Equation (4.5) denoted the RMSE.

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(y_t - \hat{y}_t)^2} \tag{4.5}$$

in which N is the number of observations of time series; y_t represents the observed value in t; $\hat{y}_i$ is the forecast of y_t.

In all cases of application of the presented measures, the best model (or optimal windows size) considers the lowest performance measures.

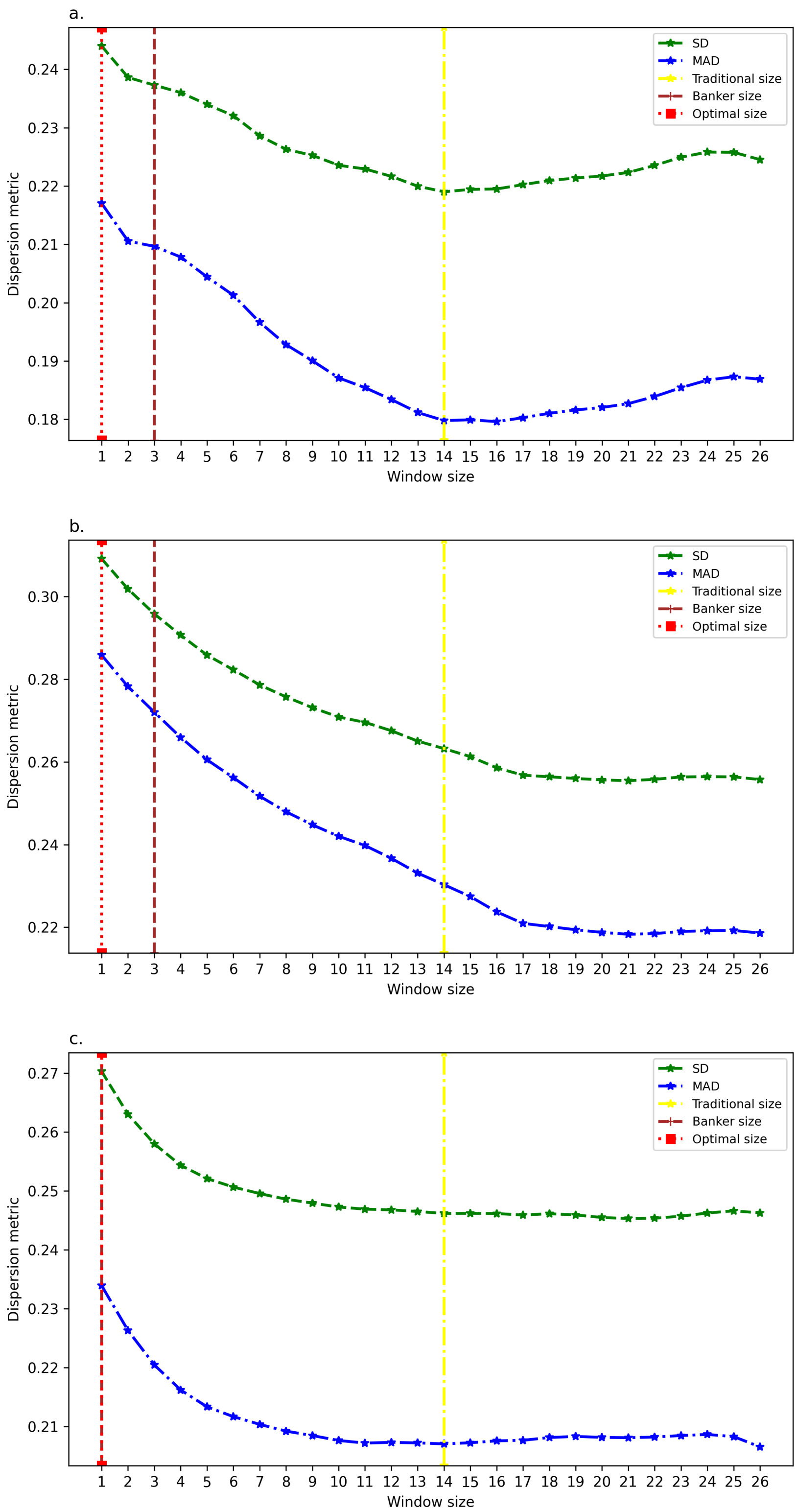
5 RESULTS

This Chapter is divided in two sections. Section 5.1 is dedicated to measuring national eco-efficiency. To obtain the eco-efficiency time series, WDEA was applied. Finally, Section 5.2 is dedicated to eco-efficiency modeling and prediction. In particular, there is a subsection with general results (Subsection 5.2.1) and another with detailed analyzes (Subsection 5.2.2).

5.1 Eco-efficiency time series

Following the proposed method (Chapter 3), national eco-efficiency is measured considering $1 \leq n \leq N$, for Mercosul, BRICS and G18. For each group, dispersion metrics (SD and MAD) is calculated. Figure 4 presents the dispersion metrics taken into account. In the three groups, there are variations in dispersion metrics. Particularly, the BRICS group recorded greater dispersion metrics between the windows in relation to Mercosul and G18. These results indicate greater divergence along the windows. MAD is less than SD in all cases. Further, as n increases, the dispersion metrics decrease. Thus, the larger the window size, the less divergence between territories. However, in all cases, window size 1 is optimal, maximizing the dispersion. Particularly, SD (MAD) maximum are 0.2440 (0.217), 0.3092 (0.2859) and 0.2703 (0.2339) for Mercosul, BRICS and G18, respectively. Thus, window size equal 1 implies eco-efficiency divergence between territories. Particularly, a greater number of benchmarks on the efficiency frontier leads to an increase in dispersion, that is, divergence (NEŽINSKỲ, 2014; LIN *et al.*, 2018). In this case, the selected window size allows highlighting of benchmarks.

Figure 4 – Eco-efficiency metrics dispersion (SD and MAD) for a. Mercosul, b. BRICS, c. G18



According to Section 2.3, the window size was also calculated with traditional and Banker rules. So, Table 4 present results of this rules. In the traditional rule, as the time series size is the same for three cases ($N = 26$), the window size is the same ($n = 14$). On the other hand, the window size calculated by Banker rule decreases with the number of DMUs. Thus, for Mercosul and BRICS, the optimal window has size equal to 3. For G18, the optimal width is 1 and coincides with that obtained considering the maximum dispersion of eco-efficiency.

Table 4 – Number of DMUs (k); window size (n) according to traditional, Banker and maximum dispersion

Group	k	Traditional rule	Banker rule	Maximum dispersion
Mercosul	5	14	3	1
BRICS	5	14	3	1
G18	18	14	1	1

Thus, in this study, a window size of 1 is considered to calculate the eco-efficiency time series in the three cases. Therefore, Section 5.2 presents the modeling and forecasting steps for eco-efficiency time series.

5.2 Eco-efficiency modeling and forecasting

This section deals with modeling and forecasting country national eco-efficiency, as well as projection. Thus, Subsection 5.2.1 presents the general results, emphasizing mainly the behavior of groups (Mercosul, BRICS and G18). In Subsection 5.2.2, the eco-efficiency of two countries in each of these groups is examined.

5.2.1 General results

After selecting the optimal window size, eco-efficiency time series were calculated considering each case (Mercosul, BRICS and G18). Then the sets are splitted into training and testing, with 75% and 25%, respectively. Therefore, given the size of the time series of 26 years, the last 6 years are intended for testing. In each group, the benchmarks were removed. Following the methodological procedure, time series with more than 50% of years with eco-efficiency equal to 1 in the training data set are removed. Thus, from Mercosul and BRICS, Brazil was removed. In the case of the G18, Brazil, Canada, Japan, France, Saudi Arabia, Italy, and the United Kingdom are benchmarks. The training data set was standardized (see Equation 4.1). Thus, ten individual time series models (ARIMA, ETS, SVR, DTR, LSTM, RFR, XGB, SM, SA, MV), and eight with pooled structure (SVR, DTR, LSTM, RFR, XGB, SM, SA, and MV) were trained. The description of the models is presented in Appendix A. In particular, the Tables 6 to 10 are for Mercosul; Tables 11 to 15 for BRICS; and 16 to 27 for G18.

From Tables 6 to 27, in Appendix A, the models description can be summarized:

- In the ARIMA model, p varies between 0 (in 14 series), 1 (in 3 series) and 2 in 1 series.
- ETS (A, N, N) was present in all series. In other words, the error is additive, without trend and seasonality.
- The best SVR hyperparameters distribute to individual time series and pooled structure as follows: C varied between 11 and 19 for individual time series; and 1 in pooled approaches; ϵ in 0.1 and 0.5; and 0.001 and 0.010, respectively; γ between 0.05 and 0.25 (individual time series); and 0.05 (pooled); *kernel* is of type `sigmoid` in 4, `rbf` in 14 `poly` in 1 for univariate time series and

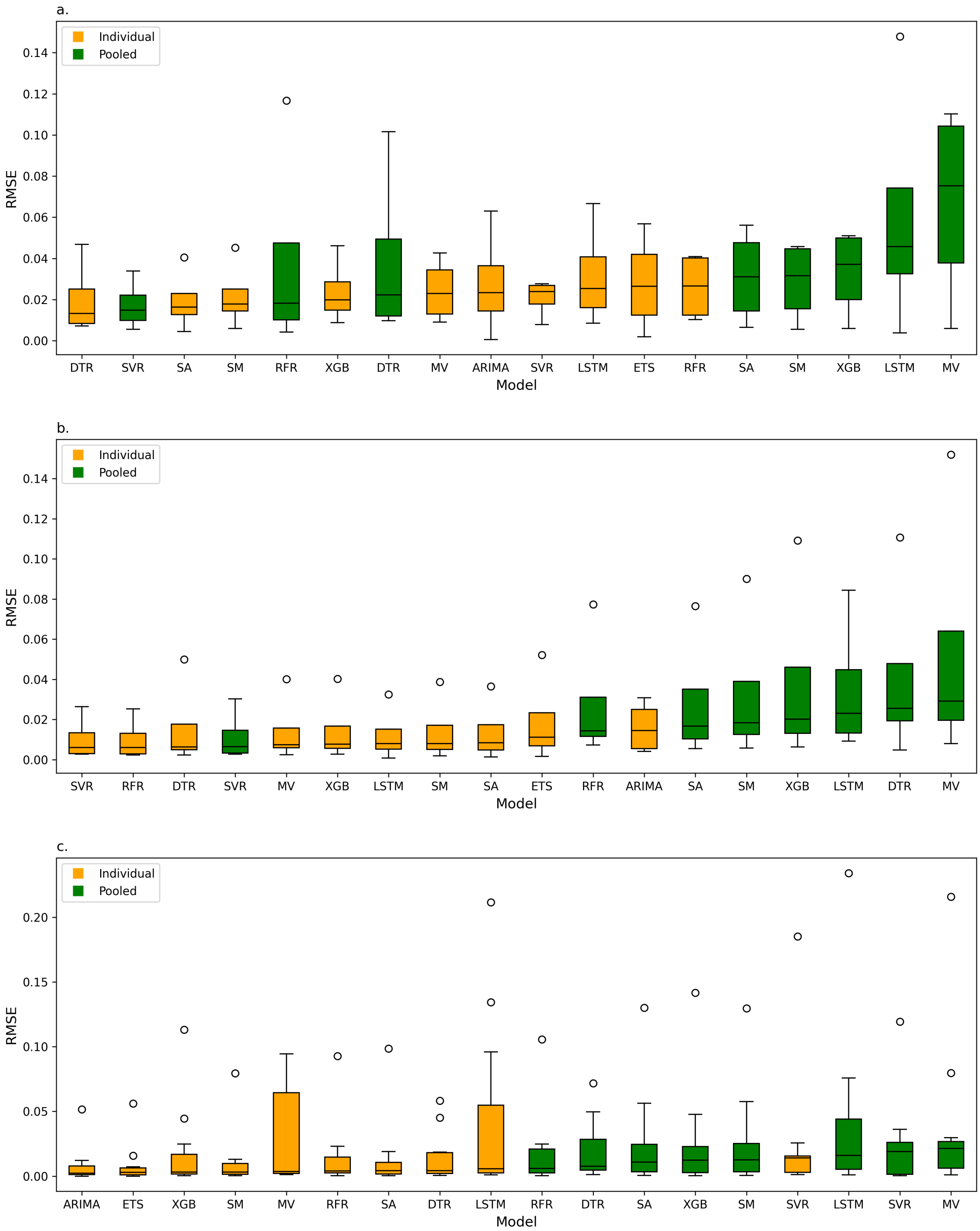
rbf for all pooled approach models.

- In DTR, max_depth for individual time series ranged between 2 and 5; and between 5 and 11 for pooled, while max_features is 11 for the first type and varied between 9 and 11 for pooled. min_samples_split varied between 8 and 12 for individual time series; and between 13 and 19 for pooled.
- For RFR constructed with individual time series, max_depth varied between 3 and 11, max_features resulted in 7, min_samples_split ranged between 5 and 12; and n_estimators between 25 and 85. In the pooled models RFR, the distribution of the best hyperparameters was: max_depth: between 7 and 14, max_features: varying between 7 and 13, min textunderscore samples_split fluctuated between 16 and 55, while n_estimators fluctuated between 180 and 450.
- In XGB for individual time series, learning_rate varied in the range between 0.005 and 0.01; max_depth fluctuated between 3 and 11, n_estimators ranged between 150 and 350, and subsample between 0.5 and 0.75. In the pooled structure, learning rate_ is 0.005, max_depth presented distribution between 7 and 14, n_estimators variations between 350 and 550, and maximum and minimum of subsample were 0.25 and 0.5 respectively.
- For the combination via MV, the weights of the ARIMA predictor varied between -3.1300 and 0.7827, while ETS were -1.2676, the minimum and 3.1508, the maximum. ARIMA and ETS predictors are only included in combinations constructed with the individual time series approach. The weights of SVR ranged between -1.9453 and 0.5446; and between 0.3846 and 0.7954 for individual time series and pooled approach, respectively. The smallest and largest weights of the DTR predictor are -0.3071 and 0.7102, and 0.0142 and 0.1656, for both approaches, in the same order. For LSTM with individual time series, the minimum weight is -0.3214 and the maximum

is 1.1678, while in the pooled structure, the weights varied between 0.0568 and 0.7409. Lastly, in XGB, the weights for the combination with individual time series ranged between -0.3946 and 1.9727, while the pooled structure was between 0.6327 and 1.3135.

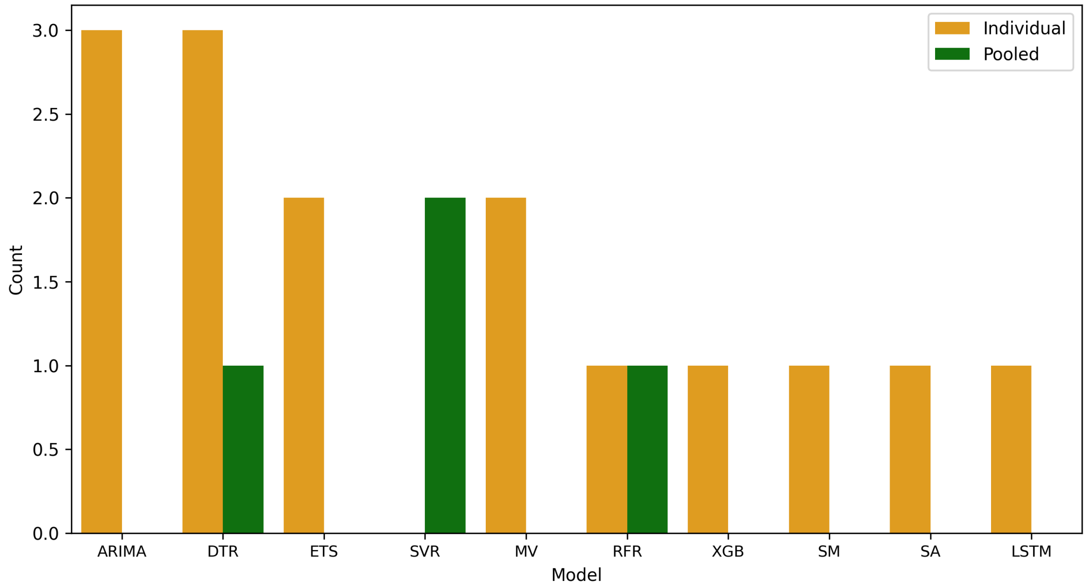
Following the methodological procedures, after the prediction considering the size of the test set, the inverse of Equation 4.1 was applied to reverse the standardization. Then, using the test data set, RMSE is calculated for the predictors of all series. Tables 29 to 30 in Appendix B present the RMSE for the three groups of countries. The figure 5 presents the distribution of performance measures for models (individual time series and grouped structure) for all cases. For Mercosul, the smallest and largest RMSE are found in Paraguay for ARIMA (0.0007) and Uruguay (0.1479) for LSTM with pooled approach, respectively. This maximum value of RMSE of LSTM is an outlier, although 50% of RMSE values are less than 0.0201 in the pooled approach. On average, RMSE of Mercosul is 0.0310. The smallest medians of RMSE are recorded for DTR with individual time series and SVR pooled, with 0.0133 and 0.0148, respectively. In the case of BRICS, the smallest RMSE has a value of 0.0008 for LSTM with individual time series (Russian Federation). On the other hand, the biggest error was for the MV predictor also for the individual series approach for China (0.1520). Although the amplitude of RMSE is smaller in relation to Mercosul (specifically models with individual time series) it presents outliers. The smallest median RMSE is recorded for SVR with individual time series (0.0060) and 0.0065 for SVR with pooled approaches. For the G18, RMSE varied between 0.0000 (ARIMA and ETS for Australia) and 0.2341 (LSTM in Turkey with individual time series). The lowest RMSE medians are for ARIMA (0.0022) and ETS (0.0028), both individual time series, while the highest are for MV (0.0214) and SVR (0.0190) adjusted with pooled approaches. As in the BRICS, RMSE for the G18 presents discrepant values. These outliers in error measures are larger for grouped approaches. In the pooled approach, only one model is trained for all series and in the case of heterogeneous series, in some of them the model tends to present greater errors in predicting the target.

Figure 5 – Distribution of models performance measures in the test data set considering individual series and the pooled structure for a. Mercosul, b. BRICS, c. G18



Based on the smallest RMSE in each time series, the winning models can be classified. Thus, Figure 6 presents the distribution of the best predictors. In this case, the count considers the three groups of countries.

Figure 6 – Distribution of the number of winning models in all time series of the three country groups considering the lowest RMSE



Considering two approaches (individual and pooled time series) all models won in at least one series. Particularly, ARIMA and DTR with individual time series wins in 3 series (15.8%) Meanwhile ETS and MV, both with individual time series, and SVR with pooled approach into 2 each (10.5%). The other models only won in one time series. Further, considering the 19 winning models, 12 (63.1%) are single models. Alongside single models, ARIMA has stood out in forecasting small series. In this study, the performance of the DTR is also verified. The study developed by Hossain *et al.* (2012) highlights applications ARIMA for forecasting the efficiency of rice crops in Bangladesh for small time series. Further, single models (e.g. Simple Moving Averages (SMA) and ARIMA) have proven to be superior to more robust models such as ANN and LightGBM in predicting time series of small-time crimes (CRUZ-NÁJERA *et al.*, 2022). In this study, therefore, as these are small series, the single models are able to better capture the behavior of the series in relation to the combined models. However, part of the literature differs in this discussion. In this sense, in the study developed by Tyralis e Papacharalampous (2017), the RFR algorithm presents better performance in forecasting one-step-ahead of short time series. In eco-efficiency time series, specifically for China, Song *et al.* (2013) found that the combination of GM and double MA is better than two independent forecast results. In a study predicting electricity demand in Indonesia with very small time series (only nine observations, from 2007 to 2015) it is found that the GM model outperforms the double MA model and Holt's exponential smoothing (KARTIKASARI; PRAYOGI, 2018). Specifically dealing with regional eco-efficiency, the Li *et al.* (2017) and Carboni e Russu (2018) used RBFN and GM, for forecasting regional energy efficiency in China and forecasting regional environmental and economic efficiency in Italy, respectively. These studies did not employ performance measures to check forecast errors. However, eco-efficiency simulations using frontier estimates are used to compare model predictions. In both cases, the predictions were close to the simulated values, with emphasis on (LI *et al.*, 2017) which mentions the performance of the RBFN-based hybrid model for predicting regional energy efficiency in China. Further, research conducted by Cerqueira *et al.* (2022) and Cerqueira *et al.* (2023) has shown that time series training sample size is relevant for predicting performance across different predictive models.

The Table 5 summarizes the number and percentage of winning models considering the lowest RMSE.

Table 5 – Number (#) and percentage (%) of series in which country groups win with the lowest RMSE for both approaches (individual and pooled time series)

Type	Count	Mercosul	BRICS	G18
Individual	#	2.00	3.00	10.00
	%	50.00	75.00	90.90
Pooled	#	2.00	1.00	1.00
	%	50.00	25.00	9.10

General results indicate in all cases, of 19 historical eco-efficiency series, the individual approach wins in 15 (68.4%), and the pooled approach in 4 (31.6%). In the case of Mercosul, models with pooled approaches win in 50% of the time series. The difference between these approaches increases in the next groups. Thus, for the BRICS, the approach with individual time series outperforms the pooled structure in 3 (75%) of the series. In G18, models with individual time series outperform the pooled approach by 90.90%. Several factors may explain differences between groups that affect the pooled approach. On average, eco-efficiency is less than 0.82 in the three groups. However, differences can be pointed out. General results indicate the Mercosul presents the largest average eco-efficiency comparing G18 and BRICS, in this order. An exception is the period from 2004 to 2009 in which Mercosul reduced eco-efficiency. In turn, the average eco-efficiency Mercosul varies from 0.46 to 0.82; For BRICS, the minimum is 0.4 and the maximum is 0.68; In G18, variations occurred between 0.52 and 0.65. Fluctuations in countries' eco-efficiency are related to their economic and environmental variables.

To recap, for eco-efficiency it is relevant that economic growth increases while environmental resources decrease. In turn, energy consumption from primary sources is directly related. According to the literature, energy is essential for the economic development of nations and must be linked to sustainable, safe and efficient strategies based on viable economic and ecological approaches for the short and long term (CAMIOTO *et al.*, 2016). Particularly, over time the G18 shows greater energy consumption, followed by the BRICS and finally Mercosul. On average, G18 countries use more than twice the energy used by Mercosul. Furthermore, over time since 1997, the energy consumption ratio between the G18 and the BRICS has decreased. BRICS countries are approaching the G18 in terms of average energy consumption. Particularly, the BRICS group involves emerging economies, i.e., developed countries. Since 1990, BRICS countries have dedicated themselves to reforms in order to promote economic development. Thus, they promoted an increase in consumer spending and industrial production (HUANG; ELING, 2013). In any case, input consumption increases, for example, energy from fossil fuels. Therefore, greater energy consumption indicates an increase in greenhouse gas emissions. In this period, BRICS and G18 present greater average greenhouse gas emissions. Particularly, average BRICS overcame G18 in 2005. On the other hand, average GDP of the BRICS is lower than the G18. Thus, increased environmental impacts with decreasing economic results can lower eco-efficiency following the classical concept of eco-efficiency (DYLLICK; HOCKERTS, 2002; KUOSMANEN; KORTELAINEN, 2005). In addition to differences between groups, countries also present differences in production factors internal to the groups. Some summary information from the World Bank (2024) and Ritchie *et al.* (2023) from 1995 to 2020 can characterize such differences between countries:

- In Mercosul, the lowest use of arable land out of total land was in Bolivia (average 3.56%) and highest in Argentina (average 12.6%). The difference increases within the other groups: in the BRICS, the Russian Federation uses 7.5%, while India uses 57.3%; for the G18, Saudi Arabia with 1.6%, against India (57.3%).
- In the case of the labor force and gross fixed capital formation in Mercosul is approximately 47% of the population and 17% of the GDP, respectively, in all countries. For labor force, in BRICS, India and China utilizing the 36.9%

and 57.4% of the total population, respectively;

- In the case of capital, South Africa used 16.62% of GDP while China more than doubled it (38.9%). Furthermore, for two G18 variables, on average, Argentina uses the lowest amount of capital, with 16.43% of GDP and Tukey the lowest percentage of employed people (36% of the labor force). In both variables in this group, China records the highest averages.
- Furthermore, energy consumption also fluctuates between countries in groups. In Mercosul, the highest average consumption is recorded in Paraguay with 22,683.82 joules of energy. Bolivia uses the lowest average (6,361.9 joules of energy). In the BRICS countries, the lowest consumption is in India, on average 4,674.9 joules of energy, and the highest is recorded in the Russian Federation, 53,227.8 joules of energy. Finally, in the G18, India also has the lowest energy consumption while Canada leads the ranking with the highest average, with 113,999.5 joules of energy.

In conclusion, the economic and environmental practices adopted by countries differ between groups. In these cases, the inherent heterogeneity of countries and their actions influences the increase in eco-efficiency, reflected in the joint approach and modeling. Thus, just one predictor simultaneously considering the country eco-efficiency time series may not capture such differences when compared to the strategy in which each series is modeled individually. Furthermore, a detailed analysis considering the countries that represent these groups may be relevant to examine national eco-efficiency. Therefore, Subsection 5.2.2 briefly discusses some results considering individual countries.

5.2.2 Detailed results

According to the previous subsection, national eco-efficiency time series of all groups (Mercosul, BRICS and G18) are modeled considering individual and pooled time series. Thus, Figures 11 to 28 in Appendix C show the forecast graphs of all models adopted. Taking the smallest RMSE, the best model is selected. Particularly, the best predictions were:

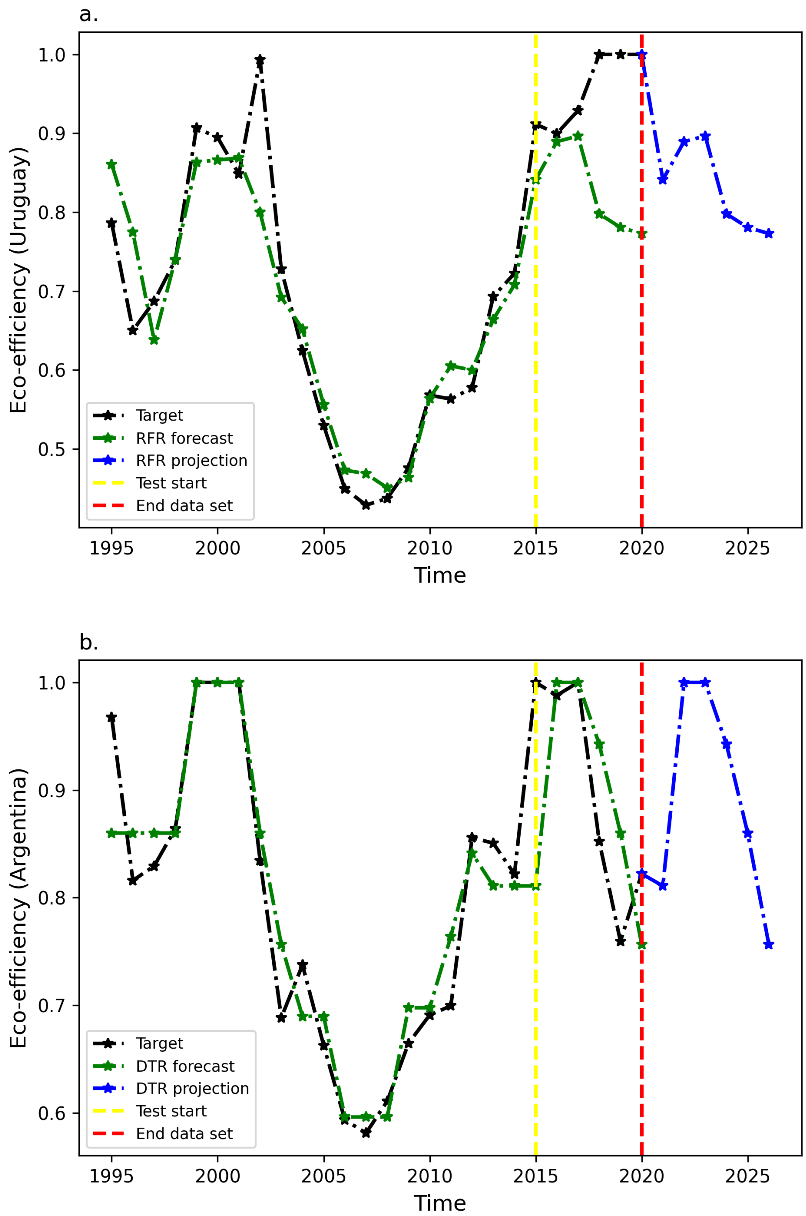
- Thus, for Mercosul, DTR and RFR won with the pooled approaches for Argentina and Uruguay, respectively; DTR and ARIMA, with inidvidual time series, for Bolivia and Paraguay, in that order.
- In the case of BRICS, the best predictors were: SVR with the pooled technique in India; RFR (China), LSTM (Russian Federation) and MV (South Africa), all considering individual time series.
- Then, in the G18 group, the following were selected, via individual time series: ARIMA (Argentina and Australia), ETS (India and South Korea), DTR (Mexico and Turkey), XGB (South Africa), SM (Indonesia), SA (China) and MV (United States). Further, the best model in Germany is SVR with pooled approach.

Utilizing the winning models in each country group time series, eco-efficiency was projected for 6-years-ahead (from 2021 to 2026). Additionally, all predictions are one-step-ahead and recursive. Therefore, in Tables 31 to 33, in Appendix D, projections of national eco-efficiency for the three groups are presented. In order to simplify the results, the average projected eco-efficiency between countries can be calculated for each group. Thus, Mercosul has an average between 0.71 and 0.80. In BRICS countries the average is approximately 0.59. These results are mainly due to the projection for China, the Russian Federation and South Africa, with low eco-efficiency values (see Table 32). Further, in the G18 the average eco-efficiency is approximately 0.58. Although 72% of G18 territories have an eco-efficiency projection above 0.5, Argentina, China and South Africa have much lower scores. Further, only the eco-efficiency projection in Australia has reached the optimal frontier ($\theta = 1$) over the years. Thus, according to Sadorsky

(2021), the G18 are an important group developed and developing that needs to show leadership in increasing eco-efficiency. In general, for three groups, the predicted average eco-efficiency is low. The groups are still far from the optimal frontier.

Thus, to represent Mercosul, Uruguay and Argentina are examined. Figure 7 presents the time series, eco-efficiency forecasts, and projections for Uruguay and Argentina (Mercosul) considering RFR and DTR, both with pooled time series, respectively.

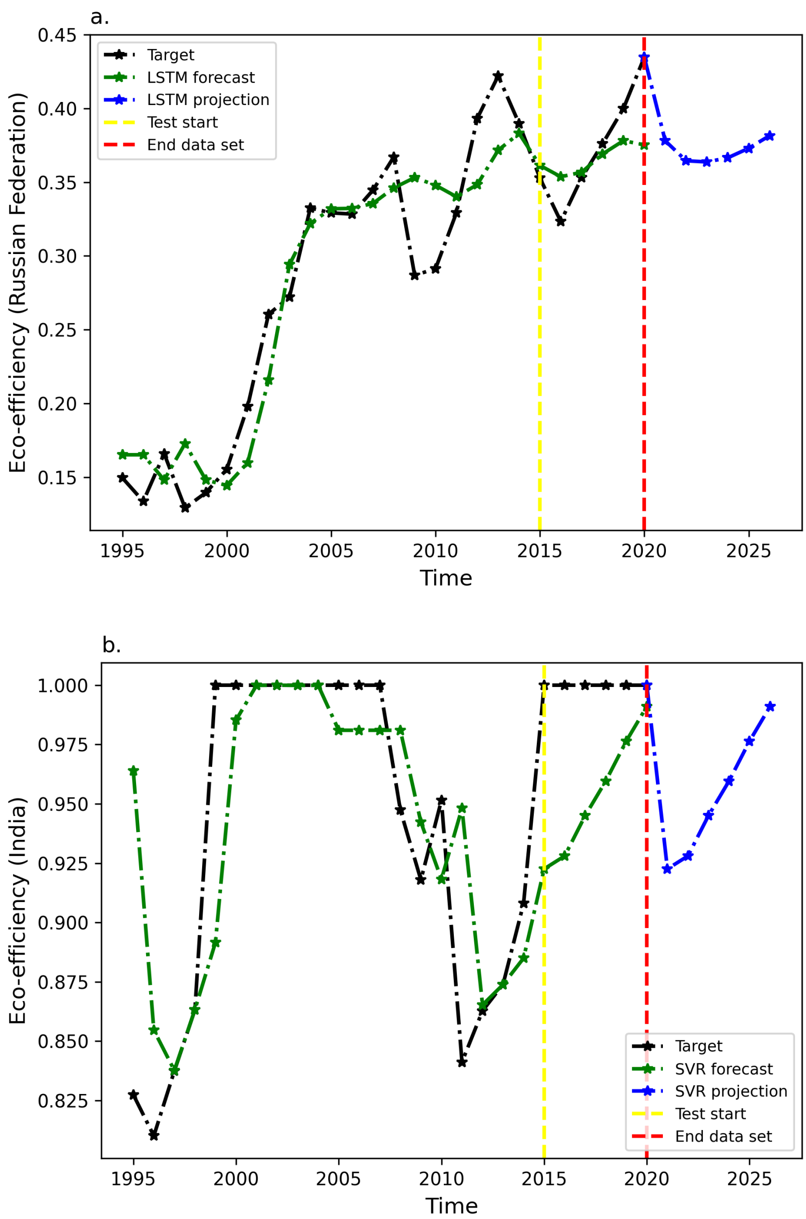
Figure 7 – Eco-efficiency time series and forecasts in Mercosul for a. Uruguay and b. Argentina



Uruguay and Argentina represent typical eco-efficiency behavior in Mercosul. In general, there were falls between 2003 and 2012 with a reverse trend in sequential years (except Paraguay). In Uruguay (Figure 7.a), given the dynamics of the series, the RFR algorithm had reasonable difficulties in tracking the test data set accurately. For eco-efficiency projections, between 2021 and 2023, Uruguay's eco-efficiency decreases from 0.84 to 0.67, then increasing in 2026 to 0.77. Furthermore, in Argentina, eco-efficiency relatively decreases in 2018 and 2019, which corresponds to the test data set. This period coincides with the growth of greenhouse gas emissions in Argentina. In 2017, Argentina reached the highest volume of emissions of this type of gas in the series studied (379,420.27 kt of CO2 equivalent) (World Bank, 2024). From Table 34 of Appendix E, it can be seen that primary energy consumption is the second variable with the greatest slack, that is, it needs to reduce on average of -41.07% for Argentina to reach the optimal frontier. In terms of projection, Argentina's eco-efficiency is expected to reach the frontier in 2022 and 2023, falling to 0.75 in 2026.

Considering BRICS countries, India presents high eco-efficiency scores variation between 0.81 and 1.0. However, the Russian Federation, China and South Africa have lower eco-efficiency scores, although similar over time. In this group, Figure 8 illustrates the time series, eco-efficiency forecasts, and projections of the Russian Federation (LSTM with individual time series) and India (SVR with pooled structure).

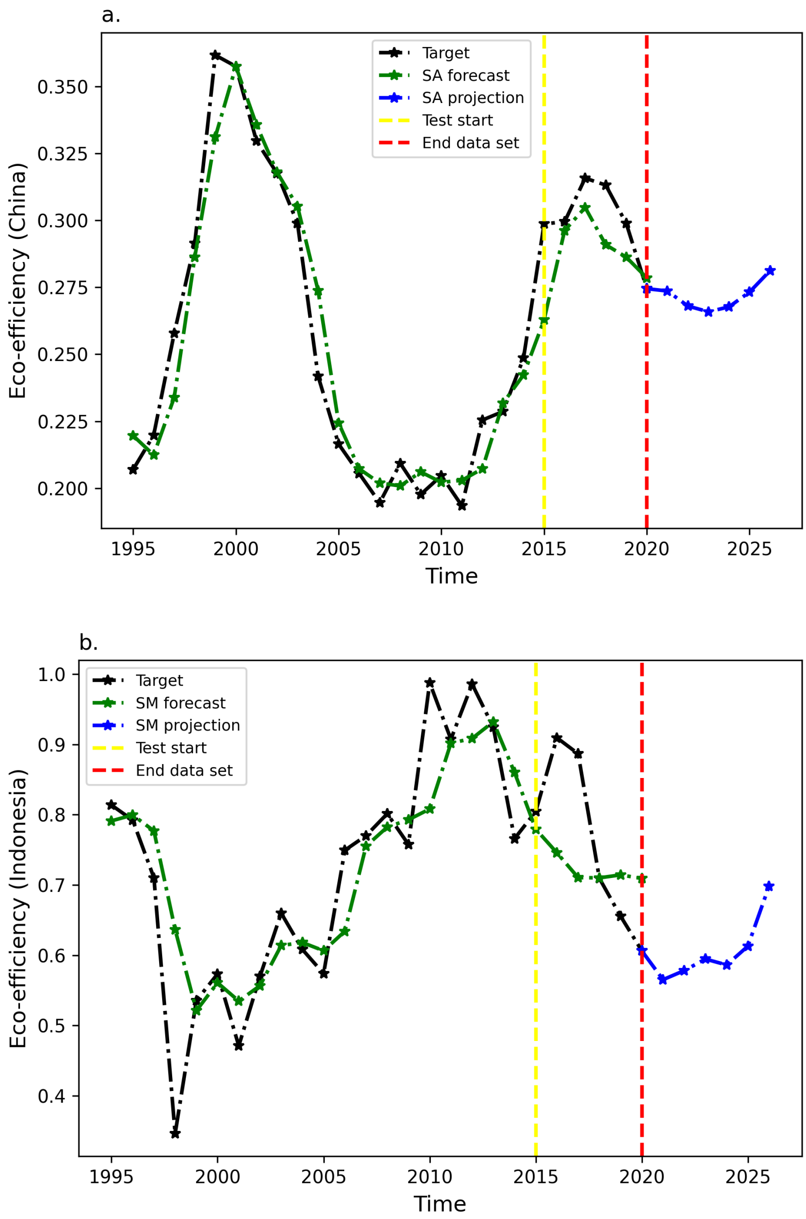
Figure 8 – Eco-efficiency time series and forecasts in BRICS for a. Russian Federation and b. India



Particularly, the Russian Federation obtained low eco-efficiency scores, with a minimum of 0.13 and a maximum of 0.43. Although with lower scores, the Russian Federation increases eco-efficiency over time. According to the study developed by (CAMIOTO *et al.*, 2016), the Russian Federation occupied the last position in the energy efficiency ranking among the BRICS. However, these authors characterize these results as a good scenario, given the fact that a few decades ago it was not a globally isolated economy. From this perspective, the progressive improvement in the eco-efficiency of the Russian Federation after 1995 may have been the result of better management of economic and environmental practices after the end of the Soviet Union. It is worth noting that in numerical terms, Russian Federation eco-efficiency is still low. And low eco-efficiency may be linked to energy consumption. In particular, according to Table 35, on average, primary energy consumption should be reduced by -91.36%. Although higher than most of the series in the BRICS, the eco-efficiency projection made by LSTM for the Russian Federation should be close to 0.38 in 2026. India's eco-efficiency in the BRICS fluctuates towards the frontier in some periods (1999 to 2007 and from 2015). Although India has high eco-efficiency, this country consumes a lot of resources. India is a country with a growing population and often prioritizes growth over environmental management (SADORSKY, 2021). In 2023, India became the most populous country in the world (World Bank, 2024). In particular, the variables with the most slack in India are arable land and gross fixed capital formation, with -41.08% and -35.05%, respectively (see Table 35 in Appendix E). However, the projection indicates that India's eco-efficiency should grow again between 2021 and 2026, reaching 0.99 in the last year (Figure 8.b).

The G18 shows greater fluctuation in eco-efficiency scores, e.g. China and South Africa record fewer values. Countries like Argentina and Australia reached the border in some periods. Thus, the time series and forecasts for China and Indonesia considering SA and SM, both with individual time series approaches are illustrated in Figure 9.

Figure 9 – Eco-efficiency time series and forecasts in G18 for a. China and b. Indonesia



China increased eco-efficiency in 1996 (maximum scores equal to 0.36) with a decrease and a new lower peak in 2017 (0.32). The eco-efficiency projection indicates that China is decreasing its eco-efficiency again, reaching 0.22 in 2026. In a study on the eco-efficiency of Chinese provinces from 2003 to 2016, Ren *et* *al.* (2020) concluded that China's overall eco-efficiency is at low levels. Further, in the research by Sadorsky (2021), China was included in the group of laggard countries, that is, those that recorded negative growth rates in eco-efficiency during the period from 1997 to 2019 and from 2019 to 2040. According to the latter author, large countries, e.g., China, tend to have intense production and weak carbon laws, which are the least efficient. In particular, China has high values for variable slacks. From the data in Table 36 of Appendix E, it can be seen that China leads the ranking of highest values for the slack in the capital (-89.85%) and labor force (-84.67%) variables.

Indonesia's eco-efficiency fluctuated during the study period. After decreasing from 0.81 to 0.35 in the period 1995 to 1998, it reversed its trajectory, approaching the optimal frontier in 2010 (eco-efficiency equal to 0.98). Then, there was a reduction in eco-efficiency reaching 0.60 in 2020. Sadorsky (2021) classified Indonesia in the group of laggard countries with negative eco-efficiency growth rates between 1997 and 2019. In the study carried out by Moutinho e Madaleno (2021a), from 2005 to 2018, Indonesia presented an average eco-efficiency of less than 87.17% in all models considered, with a variation in renewable energy consumption of -0.81%, while the Fossil fuel consumption varied by 0.09%. According to World Bank (2024) and Ritchie *et al.* (2023), in the post-2010 period until 2016, Indonesia's energy consumption ranged from approximately 71 to 72 joules of energy. In the same direction, the relationship between GDP and greenhouse gas emissions approached 1. Thus, the economic result only compensates for the environmental impact resulting from Indonesia's pollutant emissions. However, the eco-efficiency projection indicates an increase after 2021. From this perspective, in 2026 the expected value is 0.7.

In conclusion, it can be seen that the average eco-efficiency projected for the group is above all low in relation to the frontier. Specifically, the lower projection of eco-efficiency may reflect the difficulties of groups of countries in achieving targets for reducing environmental impacts, e.g., reducing pollutant emissions. It is noteworthy that the worsening of eco-efficiency is a point of impact on environmental sustainability. In other words, a large variation in eco-efficiency between countries makes it more difficult to negotiate international agreements e.g., on energy efficiency and climate change (SADORSKY, 2021). In recent history, international agreements aim to reduce harmful gas emissions. The Kyoto Protocol (1997) [1] and the Paris Agreement (2015) [2] stand out (ROBAINA-ALVES *et al.*, 2015; SUN; HUANG, 2021). In these agreements, member countries agreed to reduce emissions, that is, to reduce the risk and vulnerability of climate change. However, according to the literature, the objectives of the Paris Agreement are difficult to achieve due to the growing trend in emissions worldwide (MOR *et al.*, 2023). Added to this are the Sustainable Development Goals (SDGs) (JAMIL *et al.*, 2023). Particularly, SDGs established by the United Nations (UN), are made up of 17 goals and 169 sub-goals aimed at promoting sustainable development in all regions over the next 15 years (2016-2030) in both developing and developed countries (YANG *et al.*, 2024). Thus, the low eco-efficiency of countries can translate into the difficulty of complying with some SDGs, e.g., SDG7 (Affordable and clean energy), SDG12 (Responsible consumption and production), SDG13 (Climate action) and SDG17 (Partnerships for the goals). Therefore, low eco-efficiency scores raise warnings about the future of nations in terms of economic and environmental impacts.

[1] The Kyoto Protocol signed in Kyoto, Japan, established a reduction in pollutant emissions by at least 5% between 2008 and 2012 compared to 1990 levels (PROTOCOL, 1997).

[2] The Paris Agreement, signed in Paris, France, in 2015, aims to strengthen a global response to climate change. In particular, one goal is to keep the global average temperature rise “well below” 2 degrees Celsius above pre-industrial levels (AGREEMENT, 2015; HOROWITZ, 2016).

6 CONCLUSION

This research aimed to study a method for modeling and forecasting small time series of national eco-efficiency. Specifically, to calculate national eco-efficiency, Data Envelopment Analysis combined with Window Analysis (WDEA) was applied. Thus, in WDEA, a method based on eco-efficiency dispersion was proposed for the optimal window size computation. To model national eco-efficiency, the individual approach was used, in which each model is trained with univariate series and a pooled approach. In this second approach, the univariate time series of the territories were pooled, considering the individual effects of each country and lags. The cases studied were Mercosul, the BRICS, and the G18 countries, totaling 19 annual historical series between 1995 and 2020. Therefore, predictions were made for both approaches, RMSE was adopted to measure model errors, as well as the projection of national eco-efficiency for 6-years-ahead considering the best models. In WDEA, with maximum dispersion the optimal window size was 1 for the three groups of countries. From minimum RMSE, general results indicated the structure pooled winning in 50% of series in Mercosul, 25% in BRICS and 9.1% in G18. In turn, of 19 best models, 13 (63.1%) were single models. Particularly, for single models, ARIMA, DTR, ETS and SVR stood out. Moreover, average eco-efficiency projected for 6-years-ahead was low in the three groups.

The numerical results revealed that the pooled approach did not beat the individual time series methodology in predicting national eco-efficiency. Some implications can be examined. Firstly, the possibility of heterogeneity between countries can contribute to dissociative practices between groups. In other words, even though they are part of the same group, countries have different natural resources and production factors. In this sense, Mercosul is a more homogeneous group, while the G18, for example, brings together more heterogeneous economies.

Secondly, international agreements that require the joint participation of nations and the fulfillment of objectives may not be fully complied by countries. Thus, a hypothesis raised is that countries can modify the control of their variables resulting from internal changes that affect their long-term goals. In particular, countries sign agreements on the management of environmental factors, e.g. forest conservation and greenhouse gas emissions. These variables reflect in eco-efficiency time series in the medium and long-term. However, political-ideological actions can deliberately modify participation in agreements, making it difficult to achieve global goals. This hypothesis can be investigated in more detail. Therefore, future work may analyze these issues.

Regarding the projection of eco-efficiency, improvements are needed by countries. Given that eco-efficiency directly reflects the economic and environmental impact, low expected values may indicate that many countries have not yet achieved global goals, such as SDGs. Particularly, countries can develop individual and joint actions (given individual reality) with a view to managing their economic and environmental resources. Thus, decreasing inputs and reducing environmental impacts contributes to increasing eco-efficiency. In this sense, technology can contribute to such results. Thus, technologies can increase productivity, decreasing input uses, e.g. arable land e and labor force. Further, alternative energy sources, e.g., renewable, can replace fossil fuels. In all cases, reducing the use of production factors, for example arable land and especially energy from fossil fuels, negatively affects greenhouse gas emissions. It is known that greenhouse gas emissions constitute the main environmental impacts of countries and are directly linked to low eco-efficiency.

Finally, the general evidence of this study is that time series models applied to the prediction of eco-efficiency contribute to the temporal study of the relationship between economic results and environmental impacts of countries.

Thus, the applied pooled approaches may be better if only a grouped model of countries is better than the individual time series models for each territory. In some cases, therefore, the individual history of the countries must be considered. In the last instance, agreements adapted to the reality of countries and groups can provide ways to achieve the objectives and goals of sustainable development. Therefore, considering these results, actions for policymakers can be proposed. Firstly, the alignment of goals between countries and groups based on predicted eco-efficiency considering time series models. In this sense, eco-efficiency predictions via time series models provide a vision of eco-efficiency behavior in the future, allowing you to adjust goals to achieve projected values. In addition, global agreement strategies that consider the individual reality of countries in terms of economic and environmental resource endowments. Goals for countries must consider reality in terms of resources and factor endowments. Each country has its history, its past and therefore its policy needs to adapt to this reality. Lastly, use of technology to obtain and use sources of renewable resources that reduce greenhouse gas emissions. Together, these measures can contribute to increasing eco-efficiency and advancing sustainable development.

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APPENDIX A – MODELS DESCRIPTION

Table 6 – Description of models for eco-efficiency time series in Argentina (Mercosul)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=-0.0572
ETS	ETS(A, N, N): α=0.9999
SVR	p=5, C=19, ϵ=0.1, γ=0.05, $kernel$=`rbf`
DTR	p=5, `max_depth`=5, `max_features`=11, `min_samples_split`=8
LSTM	p=6
XGB	p=6, `learning_rate`=0.01, `max_depth`=11, `n_estimators`=350, `subsample`=0.5
RFR	p=3, `max_depth`=11, `max_features`=7, `min_samples_split`=5, `n_estimators`=25
MV	ω_{ARIMA}=-0.6066, ω_{ETS}=0.5877, ω_{SVR}=-0.4556, ω_{DTR}=-0.0812, ω_{LSTM}=0.4504, ω_{XGB}=1.1054

Table 7 – Description of models for eco-efficiency time series in Bolivia (Mercosul)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=0.0413
ETS	ETS(A, N, N): α=0.9999
SVR	p=6, C=15, ϵ=0.1, γ=0.1, $kernel$=`rbf`
DTR	p=6, `max_depth`=2, `max_features`=11, `min_samples_split`=8
LSTM	p=4
XGB	p=6, `learning_rate`=0.01, `max_depth`=3, `n_estimators`=350, `subsample`=0.5
RFR	p=6, `max_depth`=11, `max_features`=7, `min_samples_split`=5, `n_estimators`=25
MV	ω_{ARIMA}=-0.2355, ω_{ETS}=0.0801, ω_{SVR}=0.0747, ω_{DTR}=-0.1487, ω_{LSTM}=-0.0482, ω_{XGB}=1.2776

Table 8 – Description of models for eco-efficiency time series in Paraguay (Mercosul)

Formalism	Description
ARIMA	ARIMA(1, 1, 0): c=0.2313, ar_1=-0.4092
ETS	ETS(A, N, N): α=0.9999
SVR	p=3, C=11, ϵ=0.5, γ=0.1, $kernel$=`rbf`
DTR	p=3, `max_depth`=5, `max_features`=11, `min_samples_split`=9
LSTM	p=5
XGB	p=3, `learning_rate`=0.01, `max_depth`=11, `n_estimators`=350, `subsample`=0.75
RFR	p=6, `max_depth`=11, `max_features`=7, `min_samples_split`=5, `n_estimators`=25
MV	ω_{ARIMA}=0.0107, ω_{ETS}=-0.0337, ω_{SVR}=-0.0814, ω_{DTR}=-0.0119, ω_{LSTM}=0.0376, ω_{XGB}=1.0787

Table 9 – Description of models for eco-efficiency time series in Uruguay (Mercosul)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=-0.0209
ETS	ETS(A, N, N): α=0.9999
SVR	p=6, C=11, ϵ=0.1, γ=0.05, $kernel$=`rbf`
DTR	p=4, `max_depth`=5, `max_features`=11, `min_samples_split`=8
LSTM	p=5
XGB	p=5, `learning_rate`=0.01, `max_depth`=11, `n_estimators`=350, `subsample`=0.5
RFR	p=5, `max_depth`=11, `max_features`=7, `min_samples_split`=5, `n_estimators`=55
MV	ω_{ARIMA}=-0.8119, ω_{ETS}=0.6086, ω_{SVR}=-0.1729, ωDTR=0.0173, $\omega LSTM$=0.2286, ωXGB=1.1305

Table 10 – Description of models for eco-efficiency time series in Mercosul (pooled)

Formalism	Description
SVR	p=5, C=1, ϵ=0.001, γ=0.05, $kernel$=`rbf`
DTR	p=5, `max_depth`=5, `max_features`=9, `min_samples_split`=13
LSTM	p=6
XGB	p=6, `learning_rate`=0.005, `max_depth`=14, `n_estimators`=550, `subsample`=0.25
RFR	p=6, `max_depth`=14, `max_features`=7, `min_samples_split`=16, `n_estimators`=450
MV	ω_{SVR}=-0.7954, ω_{DTR}=0.1656, ω_{LSTM}=0.4867, ω_{XGB}=1.1430

Table 11 – Description of models for eco-efficiency time series in China (BRICS)

Formalism	Description
ARIMA	ARIMA(1, 1, 0): c=0.0492, ar_1=0.4053
ETS	ETS(A, N, N): α=0.9999
SVR	p=6, C=11, ϵ=0.1, γ=0.05, $kernel$=`rbf`
DTR	p=1, `max_depth`=2, `max_features`=11, `min_samples_split`=8
LSTM	p=6
XGB	p=6, `learning_rate`=0.01, `max_depth`=3, `n_estimators`=350, `subsample`=0.5
RFR	p=5, `max_depth`=3, `max_features`=7, `min_samples_split`=12, `n_estimators`=85
MV	ω_{ARIMA}=-0.0392, ω_{ETS}=0.0747, ω_{SVR}=0.0195, ω_{DTR}=-0.3071, ω_{LSTM}=-0.1192, ω_{XGB}=1.3712

Table 12 – Description of models for eco-efficiency time series in India (BRICS)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=0.0603
ETS	ETS(A, N, N): α=0.9762
SVR	p=4, C=11, ϵ=0.5, γ=0.25, $kernel$=`poly`
DTR	p=6, `max_depth`=5, `max_features`=11, `min_samples_split`=8
LSTM	p=6
XGB	p=5, `learning_rate`=0.01, `max_depth`=11, `n_estimators`=150, `subsample`=0.5
RFR	p=3, `max_depth`=3, `max_features`=7, `min_samples_split`=5, `n_estimators`=55
MV	ω_{ARIMA}=-0.2268, ω_{ETS}=-0.0108, ω_{SVR}=-0.2638, ω_{DTR}=0.1383, ω_{LSTM}=0.7263, ω_{XGB}=0.6368

Table 13 – Description of models for eco-efficiency time series in Russian Federation (BRICS)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=0.1326
ETS	ETS(A, N, N): α=0.9999
SVR	p=2, C=15, ϵ=0.1, γ=0.1, $kernel$=`sigmoid`
DTR	p=2, `max_depth`=5, `max_features`=11, `min_samples_split`=8
LSTM	p=6
XGB	p=4, `learning_rate`=0.01, `max_depth`=3, `n_estimators`=350, `subsample`=0.75
RFR	p=2, `max_depth`=3, `max_features`=7, `min_samples_split`=5, `n_estimators`=85
MV	ω_{ARIMA}=0.0900, ω_{ETS}=0.9404, ω_{SVR}=-1.0770, ω_{DTR}=0.0487, ω_{LSTM}=-0.1869, ω_{XGB}=1.1849

Table 14 – Description of models for eco-efficiency time series in South Africa (BRICS)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=-0.0677
ETS	ETS(A, N, N): α=0.9999
SVR	p=2, C=19, ϵ=0.1, γ=0.1, $kernel$=`rbf`
DTR	p=6, `max_depth`=2, `max_features`=11, `min_samples_split`=8
LSTM	p=6
XGB	p=3, `learning_rate`=0.01, `max_depth`=11, `n_estimators`=350, `subsample`=0.75
RFR	p=1, `max_depth`=3, `max_features`=7, `min_samples_split`=5, `n_estimators`=85
MV	ω_{ARIMA}=-3.1300, ω_{ETS}=3.1508, ω_{SVR}=-0.2290, ω_{DTR}=0.0884, ω_{LSTM}=0.1628, ω_{XGB}=0.9569

Table 15 – Description of models for eco-efficiency time series in BRICS (pooled)

Formalism	Description
SVR	p=6, C=1, ϵ=0.001, γ=0.05, $kernel$=`rbf`
DTR	p=4, `max_depth`=11, `max_features`=9, `min_samples_split`=13
LSTM	p=5
XGB	p=6, `learning_rate`=0.005, `max_depth`=7, `n_estimators`=550, `subsample`=0.5
RFR	p=6, `max_depth`=14, `max_features`=7, `min_samples_split`=16, `n_estimators`=180
MV	ω_{SVR}=-0.3846, ω_{DTR}=0.0142, ω_{LSTM}=0.0568, ω_{XGB}=1.3135

Table 16 – Description of models for eco-efficiency time series in Argentina (G18)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=-0.0695
ETS	ETS(A, N, N): α=0.9389
SVR	p=6, C=19, ϵ=0.1, γ=0.25, $kernel$=`rbf`
DTR	p=4, `max_depth`=2, `max_features`=11, `min_samples_split`=8
LSTM	p=6
XGB	p=6, `learning_rate`=0.01, `max_depth`=11, `n_estimators`=350, `subsample`=0.5
RFR	p=5, `max_depth`=3, `max_features`=7, `min_samples_split`=5, `n_estimators`=85
MV	ω_{ARIMA}=0.1492, ω_{ETS}=-0.2981, ω_{SVR}=0.0760, ω_{DTR}=0.1519, ω_{LSTM}=-0.3175, ω_{XGB}=1.2385

Table 17 – Description of models for eco-efficiency time series in Australia (G18)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=0.0154
ETS	ETS(A, N, N): α=0.9999
SVR	p=5, C=11, ϵ=0.5, γ=0.05, $kernel$=`rbf`
DTR	p=1, `max_depth`=5, `max_features`=11, `min_samples_split`=8
LSTM	p=6
XGB	p=1, `learning_rate`=0.005, `max_depth`=3, `n_estimators`=150, `subsample`=0.5
RFR	p=3, `max_depth`=3, `max_features`=7, `min_samples_split`=5, `n_estimators`=25
MV	ω_{ARIMA}=0.7827, ω_{ETS}=-1.2676, ω_{SVR}=0.5446, ω_{DTR}=0.7102, ω_{LSTM}=0.6247, ω_{XGB}=-0.3946

Table 18 – Description of models for eco-efficiency time series in China (G18)

Formalism	Description
ARIMA	ARIMA(1, 1, 0): c=0.0338, ar_1=0.4394
ETS	ETS(A, N, N): α=0.9999
SVR	p=6, C=11, ϵ=0.1, γ=0.1, $kernel$=`rbf`
DTR	p=2, `max_depth`=5, `max_features`=11, `min_samples_split`=8
LSTM	p=5
XGB	p=5, `learning_rate`=0.01, `max_depth`=11, `n_estimators`=350, `subsample`=0.75
RFR	p=6, `max_depth`=3, `max_features`=7, `min_samples_split`=5, `n_estimators`=25
MV	ω_{ARIMA}=-0.1006, ω_{ETS}=0.0483, ω_{SVR}=0.2085, ω_{DTR}=-0.0499, ω_{LSTM}=-0.1158, ω_{XGB}=1.0094

Table 19 – Description of models for eco-efficiency time series in Germany (G18)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=0.0002
ETS	ETS(A, N, N): α=0.8679
SVR	p=2, C=11, ϵ=0.1, γ=0.25, $kernel$=`rbf`
DTR	p=6, `max_depth`=2, `max_features`=11, `min_samples_split`=8
LSTM	p=6
XGB	p=2, `learning_rate`=0.01, `max_depth`=3, `n_estimators`=350, `subsample`=0.5
RFR	p=1, `max_depth`=3, `max_features`=7, `min_samples_split`=12, `n_estimators`=25
MV	ω_{ARIMA}=-0.1736, ω_{ETS}=0.0549, ω_{SVR}=-0.1406, ω_{DTR}=-0.0707, ω_{LSTM}=0.2877, ω_{XGB}=1.0422

Table 20 – Description of models for eco-efficiency time series in India (G18)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=-0.1078
ETS	ETS(A, N, N): α=0.8294
SVR	p=5, C=19, ϵ=0.5, γ=0.1, $kernel$=rbf
DTR	p=5, max_depth=2, max_features=11, min_samples_split=9
LSTM	p=6
XGB	p=6, learning_rate=0.01, max_depth=3, n_estimators=150, subsample=0.5
RFR	p=1, max_depth=3, max_features=7, min_samples_split=12, n_estimators=25
MV	ω_{ARIMA}=0.0090, ω_{ETS}=-0.2081, ω_{SVR}=0.0910, ω_{DTR}=0.1062, ω_{LSTM}=0.7834, ω_{XGB}=0.2185

Table 21 – Description of models for eco-efficiency time series in Indonesia (G18)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=-0.015
ETS	ETS(A, N, N): α=0.7112
SVR	p=5, C=11, ϵ=0.1, γ=0.05, $kernel$=sigmoid
DTR	p=1, max_depth=2, max_features=11, min_samples_split=12
LSTM	p=5
XGB	p=6, learning_rate=0.01, max_depth=11, n_estimators=350, subsample=0.5
RFR	p=1, max_depth=3, max_features=7, min_samples_split=12, n_estimators=55
MV	ω_{ARIMA}=-0.0988, ω_{ETS}=0.1190, ω_{SVR}=-0.1171, ω_{DTR}=-0.0504, ω_{LSTM}=0.5640, ω_{XGB}=0.5834

Table 22 – Description of models for eco-efficiency time series in South Korea (G18)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=0.1145
ETS	ETS(A, N, N): α=0.9999
SVR	p=5, C=11, ϵ=0.1, γ=0.1, $kernel$=rbf
DTR	p=2, max_depth=5, max_features=11, min_samples_split=9
LSTM	p=6
XGB	p=4, learning_rate=0.01, max_depth=11, n_estimators=350, subsample=0.75
RFR	p=2, max_depth=3, max_features=7, min_samples_split=12, n_estimators=85
MV	ω_{ARIMA}=-0.8455, ω_{ETS}=0.8066, ω_{SVR}=0.0053, ω_{DTR}=-0.0448, ω_{LSTM}=-0.1277, ω_{XGB}=1.2060

Table 23 – Description of models for eco-efficiency time series in Mexico (G18)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=0.0463
ETS	ETS(A, N, N): α=0.9999
SVR	p=5, C=11, ϵ=0.1, γ=0.05, $kernel$=`rbf`
DTR	p=6, `max_depth`=2, `max_features`=11, `min_samples_split`=8
LSTM	p=4
XGB	p=4, `learning_rate`=0.01, `max_depth`=11, `n_estimators`=350, `subsample`=0.75
RFR	p=6, `max_depth`=11, `max_features`=7, `min_samples_split`=5, `n_estimators`=25
MV	ω_{ARIMA}=-0.1893, ω_{ETS}=0.1225, ω_{SVR}=-0.0037, ω_{DTR}=0.0014, ω_{LSTM}=0.0836, ω_{XGB}=0.9854

Table 24 – Description of models for eco-efficiency time series in South Africa (G18)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=0.0303
ETS	ETS(A, N, N): α=0.9999
SVR	p=6, C=11, ϵ=0.1, γ=0.1, $kernel$=`rbf`
DTR	p=5, `max_depth`=5, `max_features`=11, `min_samples_split`=8
LSTM	p=6
XGB	p=4, `learning_rate`=0.01, `max_depth`=11, `n_estimators`=150, `subsample`=0.75
RFR	p=4, `max_depth`=11, `max_features`=7, `min_samples_split`=5, `n_estimators`=85
MV	ω_{ARIMA}=-0.3829, ω_{ETS}=0.3810, ω_{SVR}=-0.2498, ω_{DTR}=0.0201, ω_{LSTM}=1.1678, ω_{XGB}=0.0637

Table 25 – Description of models for eco-efficiency time series in Turkey (G18)

Formalism	Description
ARIMA	ARIMA(0, 1, 0): c=0.0001
ETS	ETS(A, N, N): α=0.9221
SVR	p=6, C=11, ϵ=0.5, γ=0.05, $kernel$=`sigmoid`
DTR	p=6, `max_depth`=5, `max_features`=11, `min_samples_split`=8
LSTM	p=6
XGB	p=6, `learning_rate`=0.01, `max_depth`=11, `n_estimators`=150, `subsample`=0.5
RFR	p=4, `max_depth`=3, `max_features`=7, `min_samples_split`=12, `n_estimators`=25
MV	ω_{ARIMA}=0.0865, ω_{ETS}=-0.0615, ω_{SVR}=-0.3532, ω_{DTR}=0.4304, ω_{LSTM}=0.5041, ω_{XGB}=0.3936

Table 26 – Description of models for eco-efficiency time series in United States (G18)

Formalism	Description
ARIMA	ARIMA(2, 1, 0): c=0.2799, ar_1=0.1678, ar_2=-0.7289
ETS	ETS(A, N, N): α=0.9999
SVR	p=1, C=15, ϵ=0.1, γ=0.05, $kernel$=`sigmoid`
DTR	p=1, `max_depth`=5, `max_features`=11, `min_samples_split`=8
LSTM	p=6
XGB	p=4, `learning_rate`=0.01, `max_depth`=11, `n_estimators`=150, `subsample`=0.5
RFR	p=2, `max_depth`=3, `max_features`=7, `min_samples_split`=5, `n_estimators`=85
MV	ω_{ARIMA}=-0.1765, ω_{ETS}=1.1807, ω_{SVR}=-1.9453, ω_{DTR}=0.2897, ω_{LSTM}=-0.3214, ω_{XGB}=1.9727

Table 27 – Description of models for eco-efficiency time series in G18 (pooled)

Formalism	Description
SVR	p=6, C=1, ϵ=0.01, γ=0.05, $kernel$=`rbf`
DTR	p=3, `max_depth`=5, `max_features`=15, `min_samples_split`=19
LSTM	p=6
XGB	p=6 `learning_rate`=0.005, `max_depth`=14, `n_estimators`=350, `subsample`=0.25
RFR	p=6, `max_depth`=7, `max_features`=13, `min_samples_split`=55, `n_estimators`=180
MV	ω_{SVR}=-0.4977, ω_{DTR}=0.1242, ω_{LSTM}=0.7409, ω_{XGB}=0.6327

APPENDIX B – FORECAST PERFORMANCE METRICS

Table 28 – RMSE for model prediction in the test data set for Mercosul (1995-2020)

Country	Type	ARIMA	ETS	SVR	DTR	LSTM	XGB	RFR	SM	SA	MV
ARG	Individual	0.0191	0.0160	0.0212	0.0179	0.0186	0.0169	0.0132	0.0172	0.0172	0.0143
	Pooled			0.0183	0.0098	0.0421	0.0247	0.0122	0.0188	0.0172	0.0484
BOL	Individual	0.0277	0.0370	0.0277	0.0088	0.0322	0.0229	0.0409	0.0185	0.0156	0.0317
	Pooled			0.0114	0.1016	0.0496	0.0511	0.1167	0.0457	0.0448	0.1102
PRY	Individual	0.0007	0.0019	0.0079	0.0072	0.0085	0.0088	0.0103	0.0060	0.0045	0.0091
	Pooled			0.0056	0.0127	0.0038	0.0059	0.0042	0.0056	0.0066	0.0060
URY	Individual	0.0630	0.0568	0.0267	0.0468	0.0667	0.0462	0.0401	0.0452	0.0405	0.0426
	Pooled			0.0339	0.0320	0.1479	0.0496	0.0244	0.0444	0.0562	0.1023

Table 29 – RMSE for model prediction in the test data set for BRICS (1995-2020)

Country	Type	ARIMA	ETS	SVR	DTR	LSTM	XGB	RFR	SM	SA	MV
CHN	Individual	0.0308	0.0522	0.0263	0.0500	0.0325	0.0402	0.0252	0.0387	0.0366	0.0401
	Pooled			0.0303	0.1107	0.0844	0.1091	0.0774	0.0901	0.0765	0.1520
IND	Individual	0.0060	0.0086	0.0031	0.0058	0.0066	0.0066	0.0031	0.0062	0.0059	0.0077
	Pooled			0.0027	0.0242	0.0145	0.0153	0.0157	0.0148	0.0120	0.0236
RUS	Individual	0.0041	0.0016	0.0028	0.0024	0.0008	0.0027	0.0024	0.0018	0.0013	0.0071
	Pooled			0.0095	0.0268	0.0316	0.0251	0.0130	0.0220	0.0214	0.0348
ZAF	Individual	0.0231	0.0138	0.0090	0.0069	0.0094	0.0088	0.0091	0.0099	0.0110	0.0024
	Pooled			0.0035	0.0048	0.0092	0.0063	0.0073	0.0058	0.0055	0.0080

Table 30 – RMSE for model prediction in the test data set for G18 (1995-2020)

Country	Type	ARIMA	ETS	SVR	DTR	LSTM	XGB	RFR	SM	SA	MV
ARG	Individual	0.0050	0.0071	0.0157	0.0091	0.0071	0.0074	0.0106	0.0074	0.0076	0.0077
	Pooled			0.0205	0.0068	0.0582	0.0100	0.0064	0.0147	0.0168	0.0296
AUS	Individual	0.0000	0.0000	0.0045	0.0583	0.0052	0.0031	0.0036	0.0017	0.0042	0.0408
	Pooled			0.0014	0.0062	0.0028	0.0019	0.0011	0.0017	0.0023	0.0037
CHN	Individual	0.0011	0.0028	0.0011	0.0040	0.0010	0.0008	0.0040	0.0004	0.0003	0.0011
	Pooled			0.0012	0.0077	0.0079	0.0035	0.0037	0.0050	0.0043	0.0102
DEU	Individual	0.0017	0.0016	0.0015	0.0020	0.0021	0.0012	0.0013	0.0017	0.0014	0.0014
	Pooled			0.0006	0.0013	0.0009	0.0008	0.0011	0.0009	0.0008	0.0011
IND	Individual	0.0103	0.0046	0.0257	0.0175	0.0960	0.0247	0.0231	0.0130	0.0138	0.0913
	Pooled			0.0361	0.0717	0.0758	0.0477	0.0248	0.0575	0.0562	0.0797
IDN	Individual	0.0121	0.0157	0.0151	0.0187	0.1343	0.0445	0.0187	0.0121	0.0190	0.0879
	Pooled			0.0291	0.0320	0.0300	0.0197	0.0213	0.0276	0.0262	0.0238
KOR	Individual	0.0013	0.0002	0.0154	0.0005	0.0136	0.0021	0.0005	0.0010	0.0021	0.0035
	Pooled			0.0054	0.0188	0.0160	0.0123	0.0059	0.0125	0.0109	0.0214
MEX	Individual	0.0054	0.0028	0.0037	0.0011	0.0057	0.0027	0.0035	0.0031	0.0030	0.0025
	Pooled			0.0190	0.0029	0.0089	0.0138	0.0040	0.0092	0.0080	0.0084
ZAF	Individual	0.0005	0.0004	0.0021	0.0020	0.0018	0.0002	0.0004	0.0006	0.0008	0.0020
	Pooled			0.0002	0.0011	0.0011	0.0004	0.0003	0.0004	0.0005	0.0011
TUR	Individual	0.0516	0.0562	0.1852	0.0450	0.2116	0.1131	0.0926	0.0794	0.0986	0.0944
	Pooled			0.1192	0.0495	0.2341	0.1416	0.1056	0.1296	0.1300	0.2157
USA	Individual	0.0022	0.0056	0.0141	0.0043	0.0027	0.0091	0.0043	0.0052	0.0055	0.0022
	Pooled			0.0231	0.0248	0.0195	0.0258	0.0204	0.0229	0.0229	0.0221

APPENDIX C – FORECAST GRAPHS

Figure 10 – Forecasts for Argentina (Mercosul) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

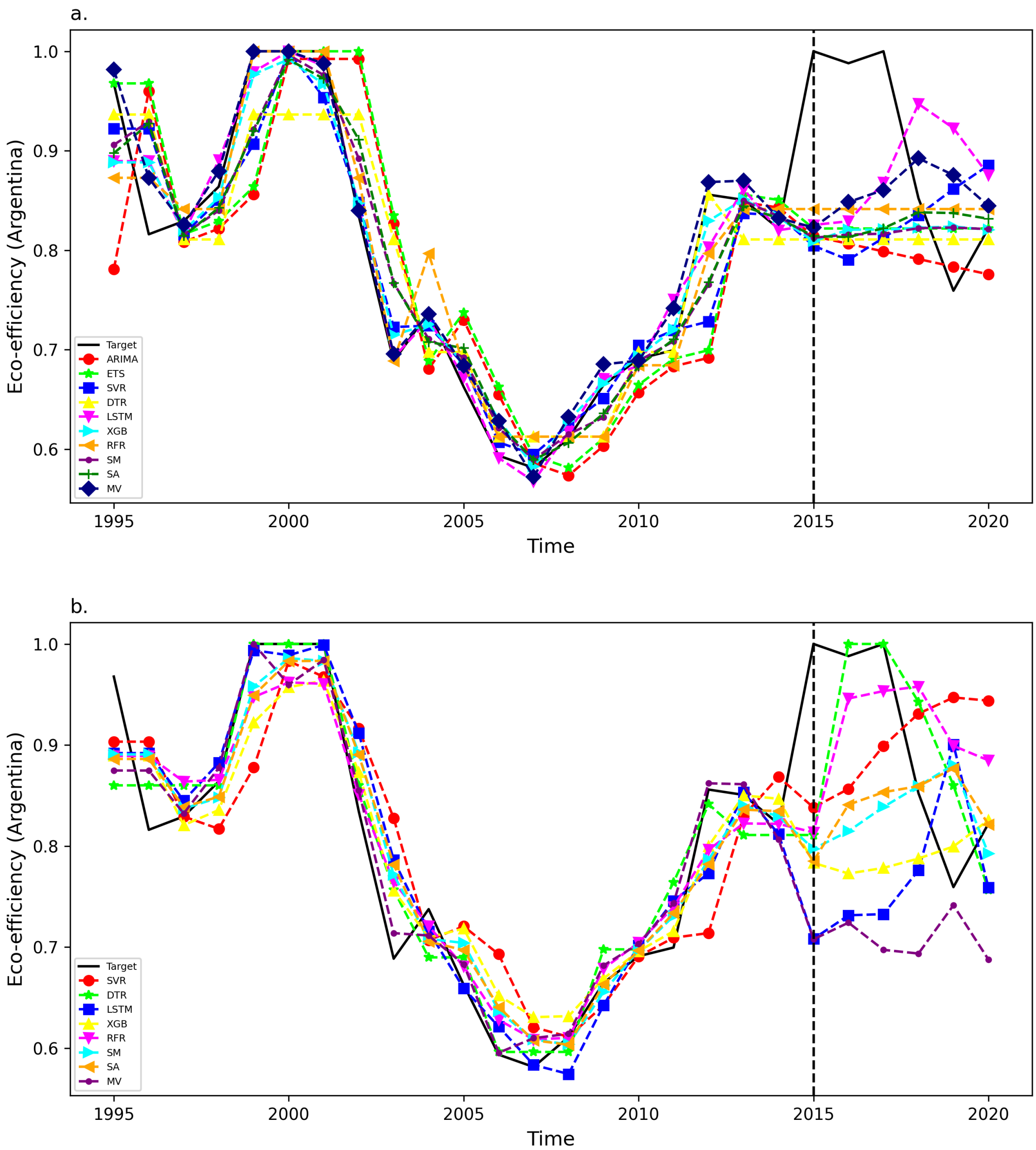


Figure 11 – Forecasts for Bolivia (Mercosul) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

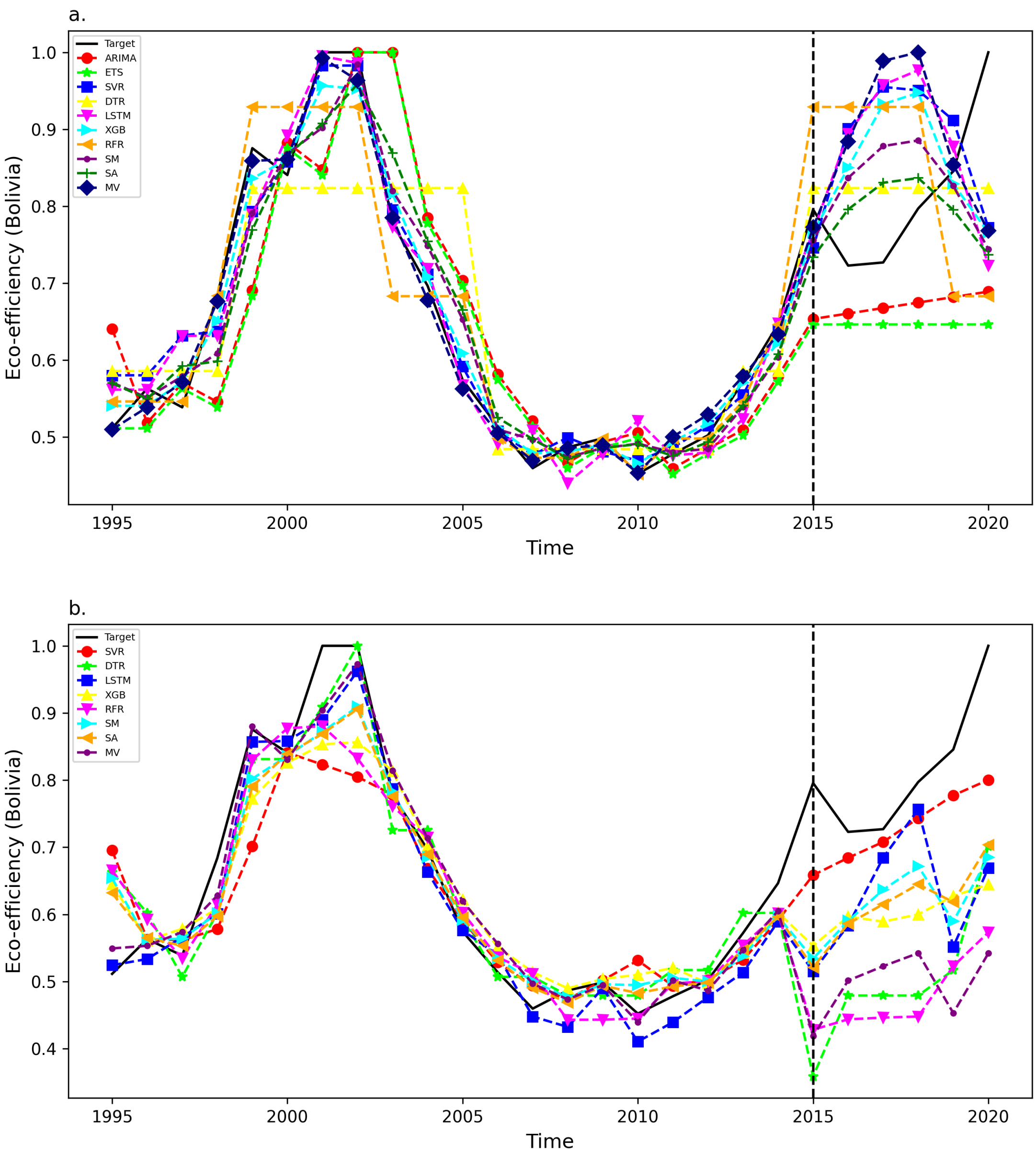


Figure 12 – Forecasts for Paraguay (Mercosul) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

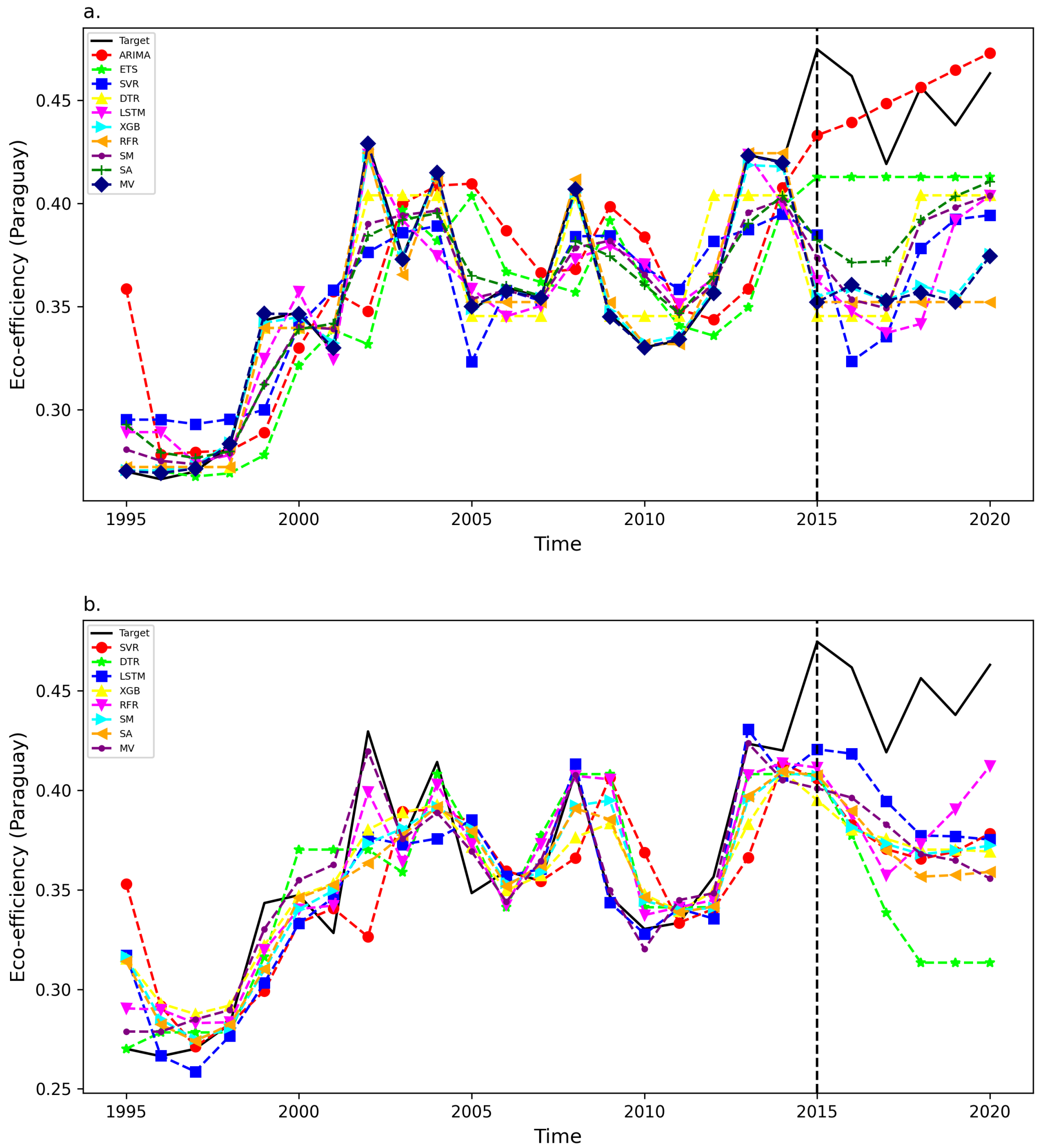


Figure 13 – Forecasts for Uruguay (Mercosul) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

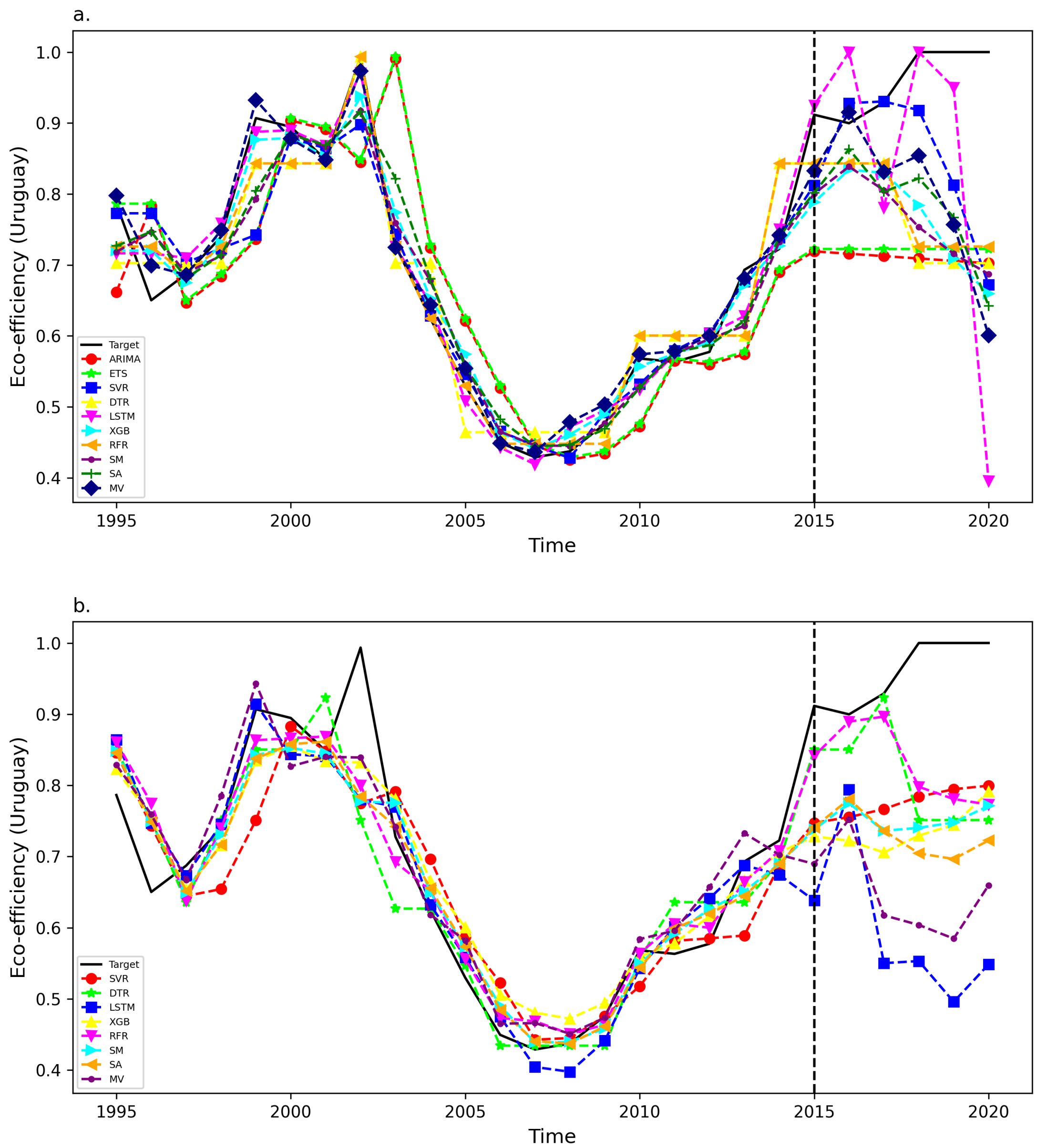
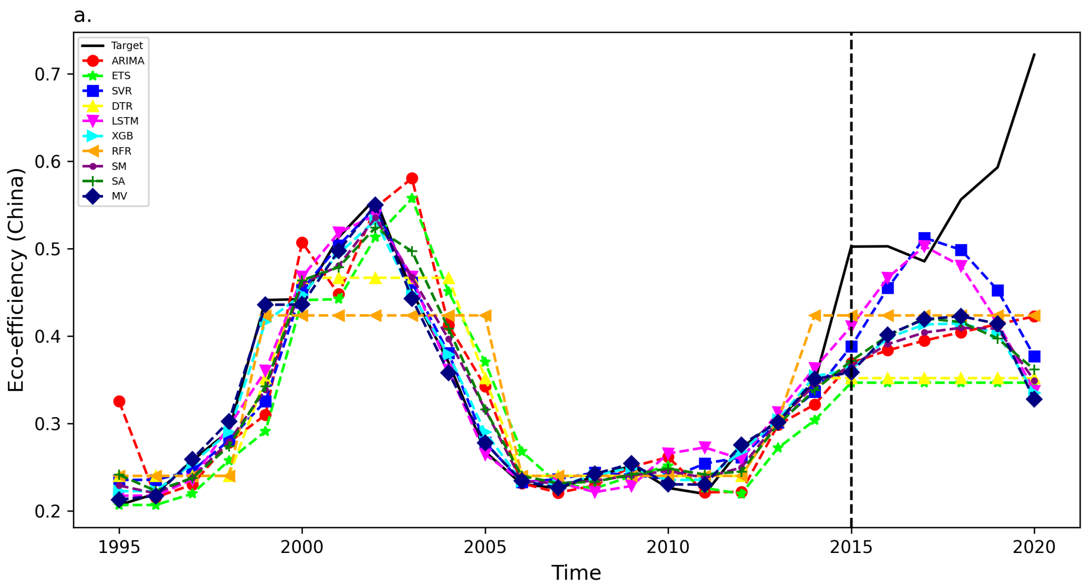


Figure 14 – Forecasts for China (BRICS) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)



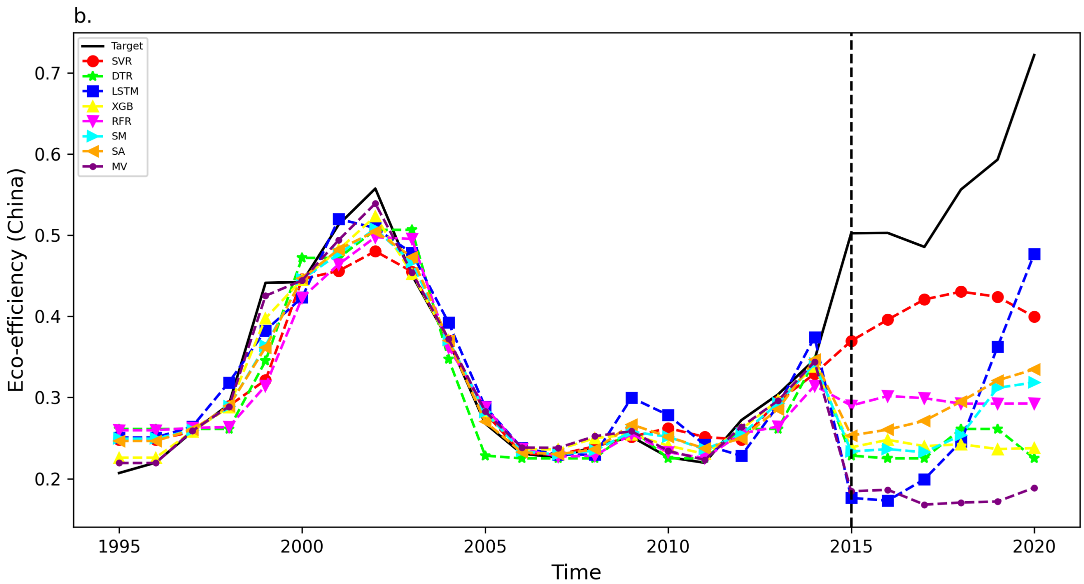
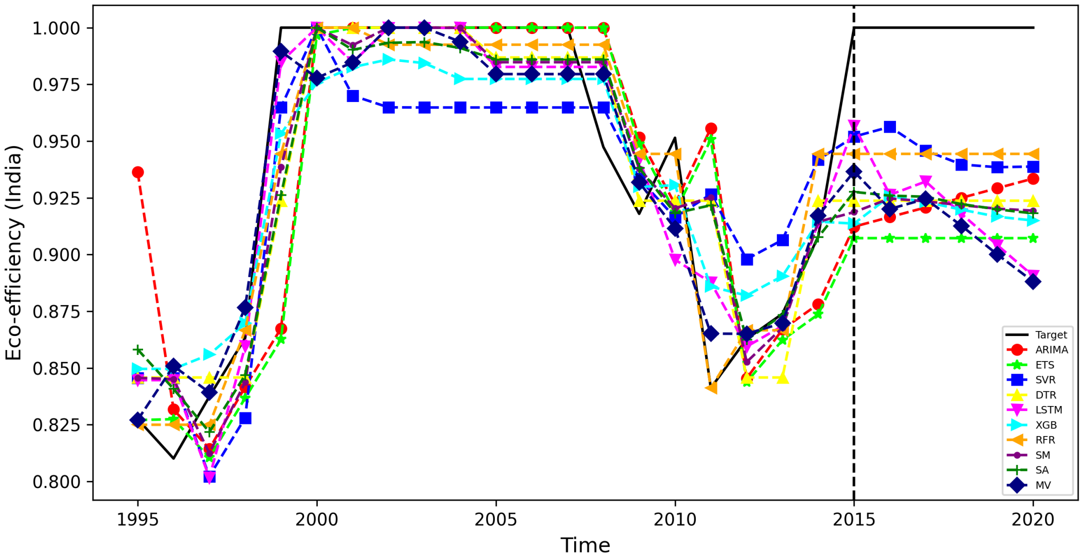


Figure 15 – Forecasts for India (BRICS) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

a.



b.

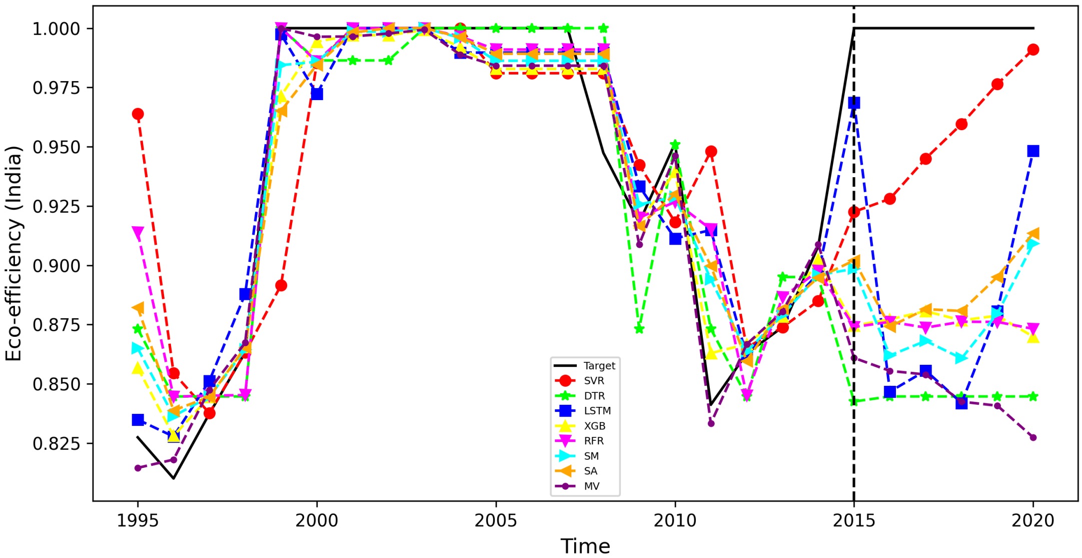
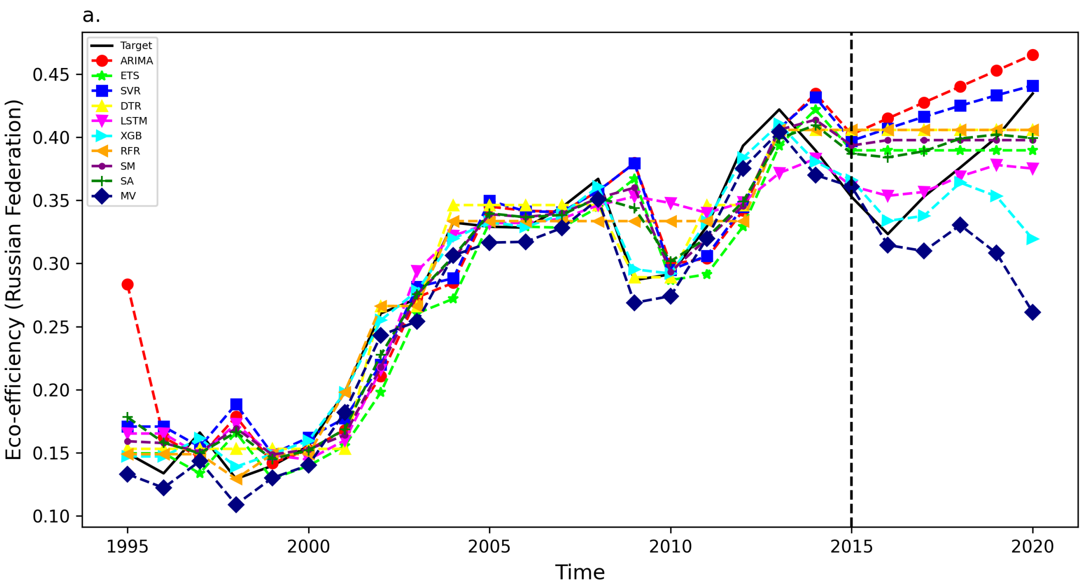


Figure 16 – Forecasts for Russian Federation (BRICS) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)



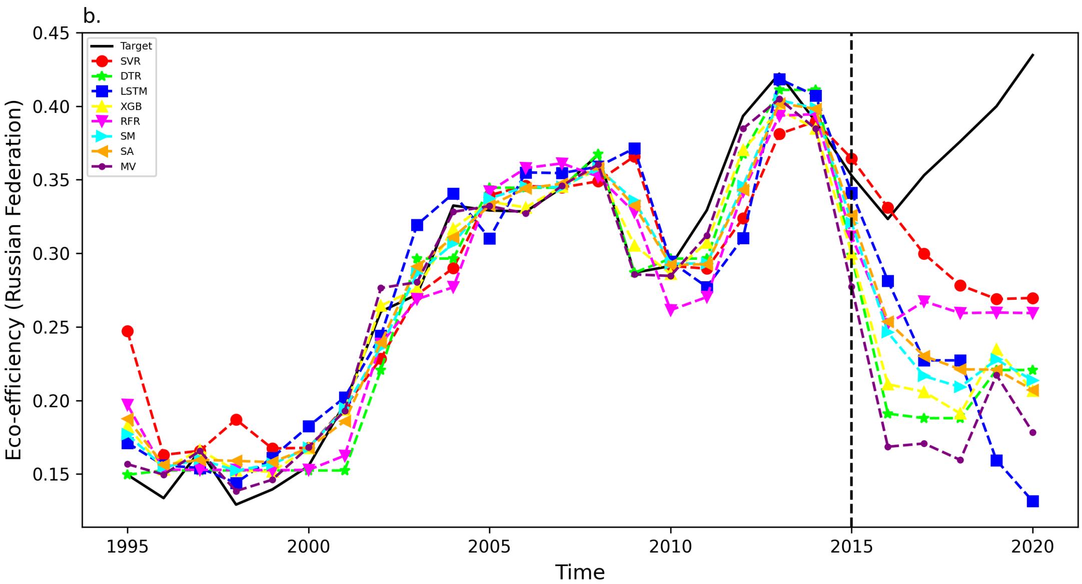


Figure 17 – Forecasts for South Africa (BRICS) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

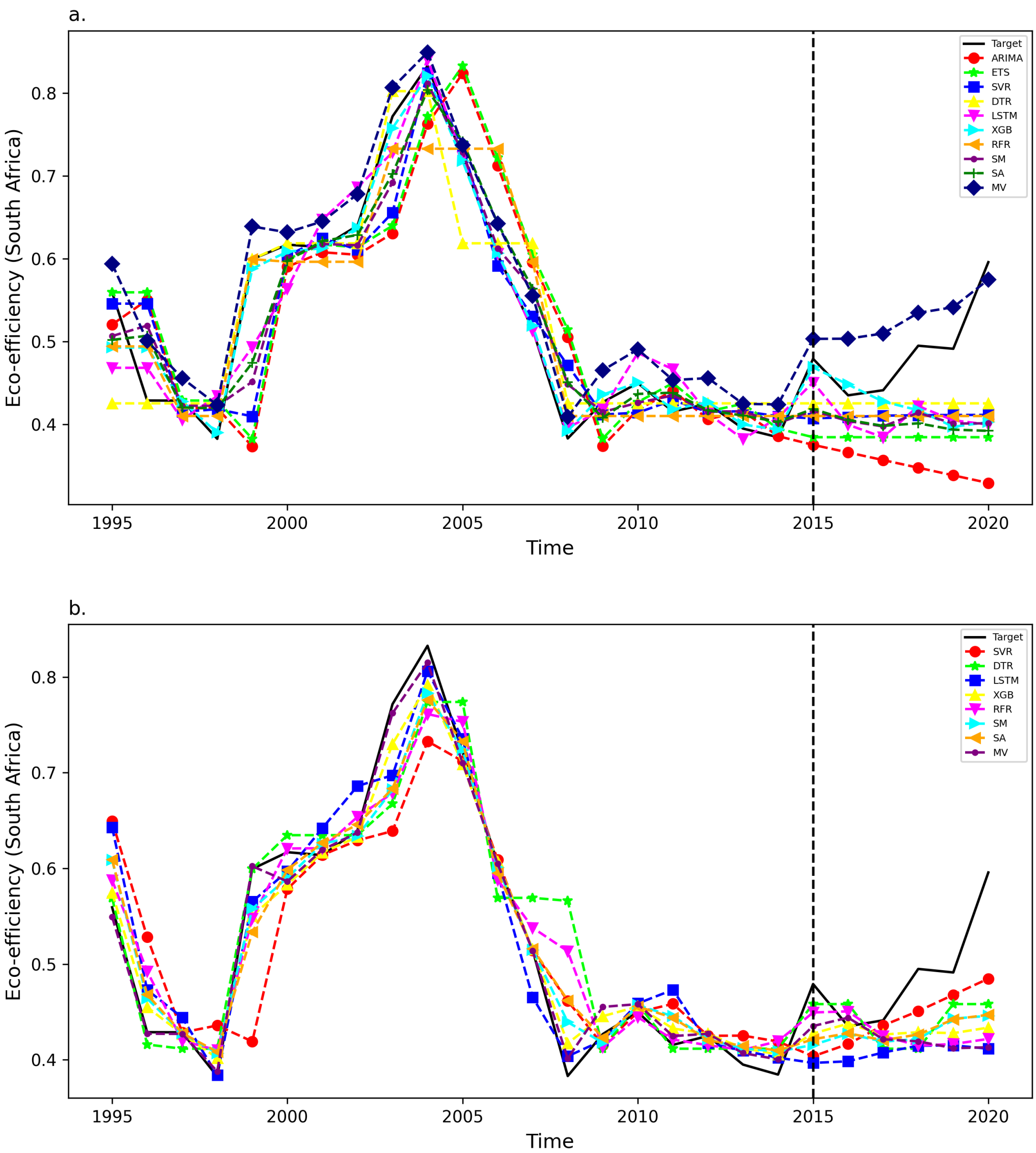


Figure 18 – Forecasts for Argentina (G18) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

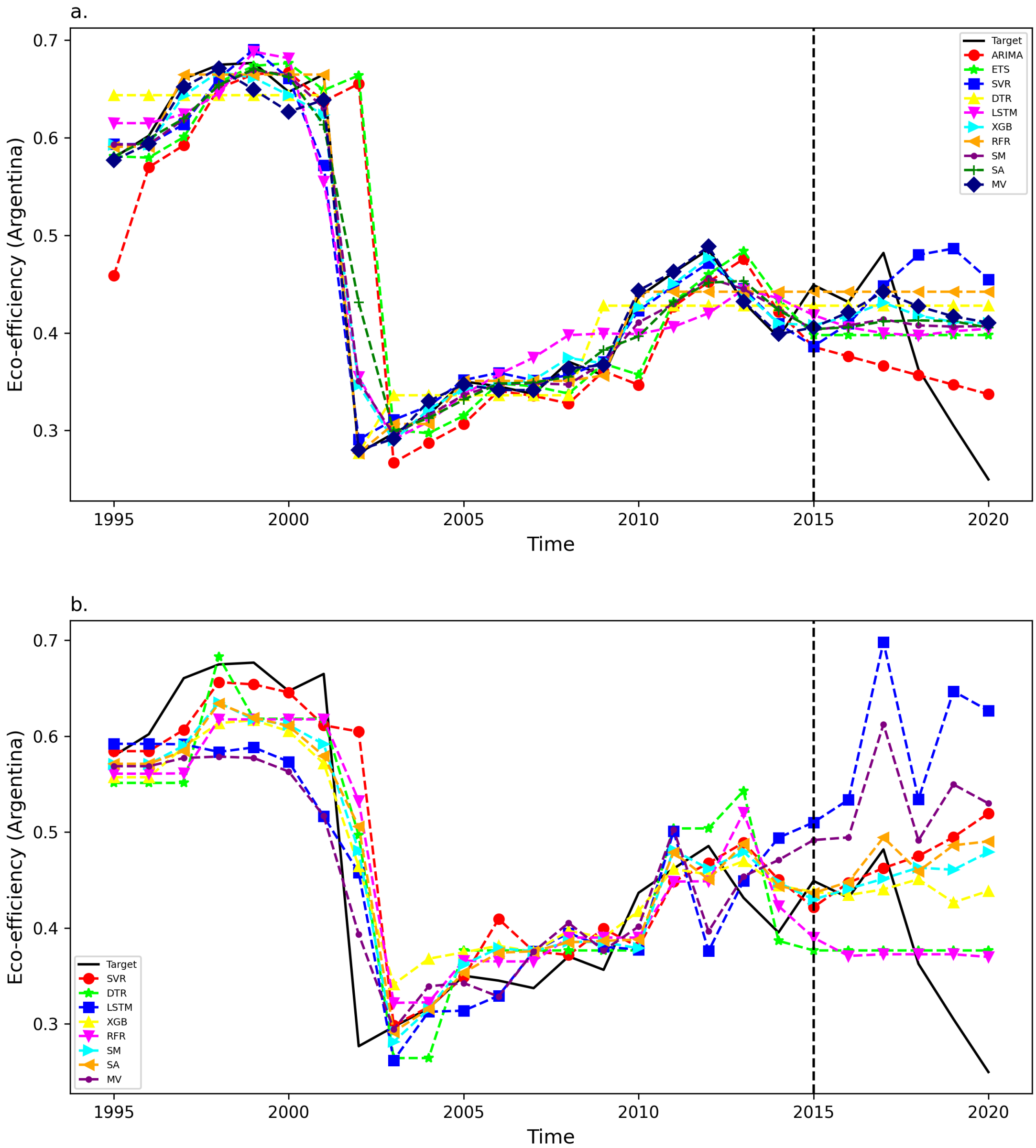


Figure 19 – Forecasts for Australia (G18) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

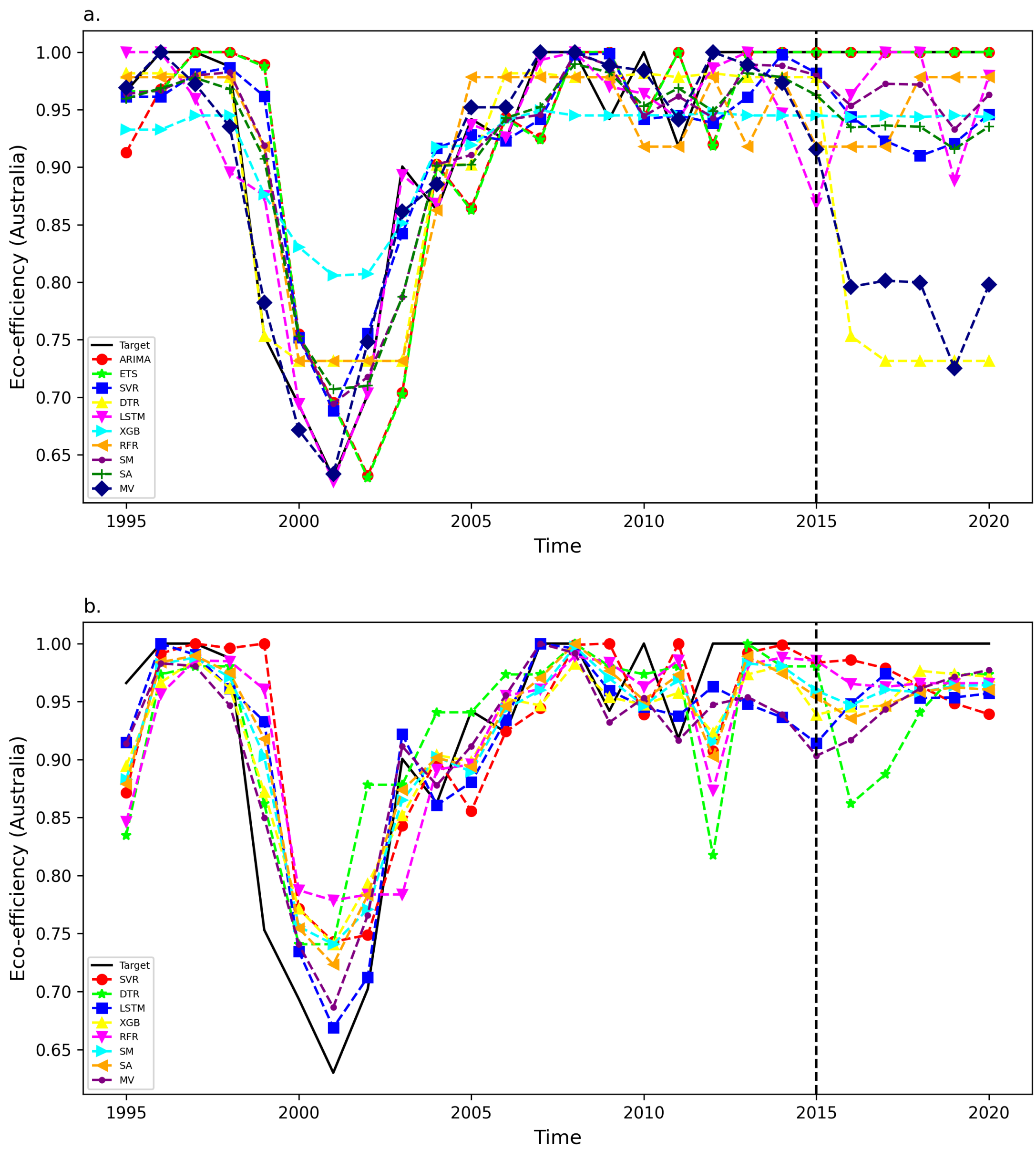


Figure 20 – Forecasts for China (G18) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

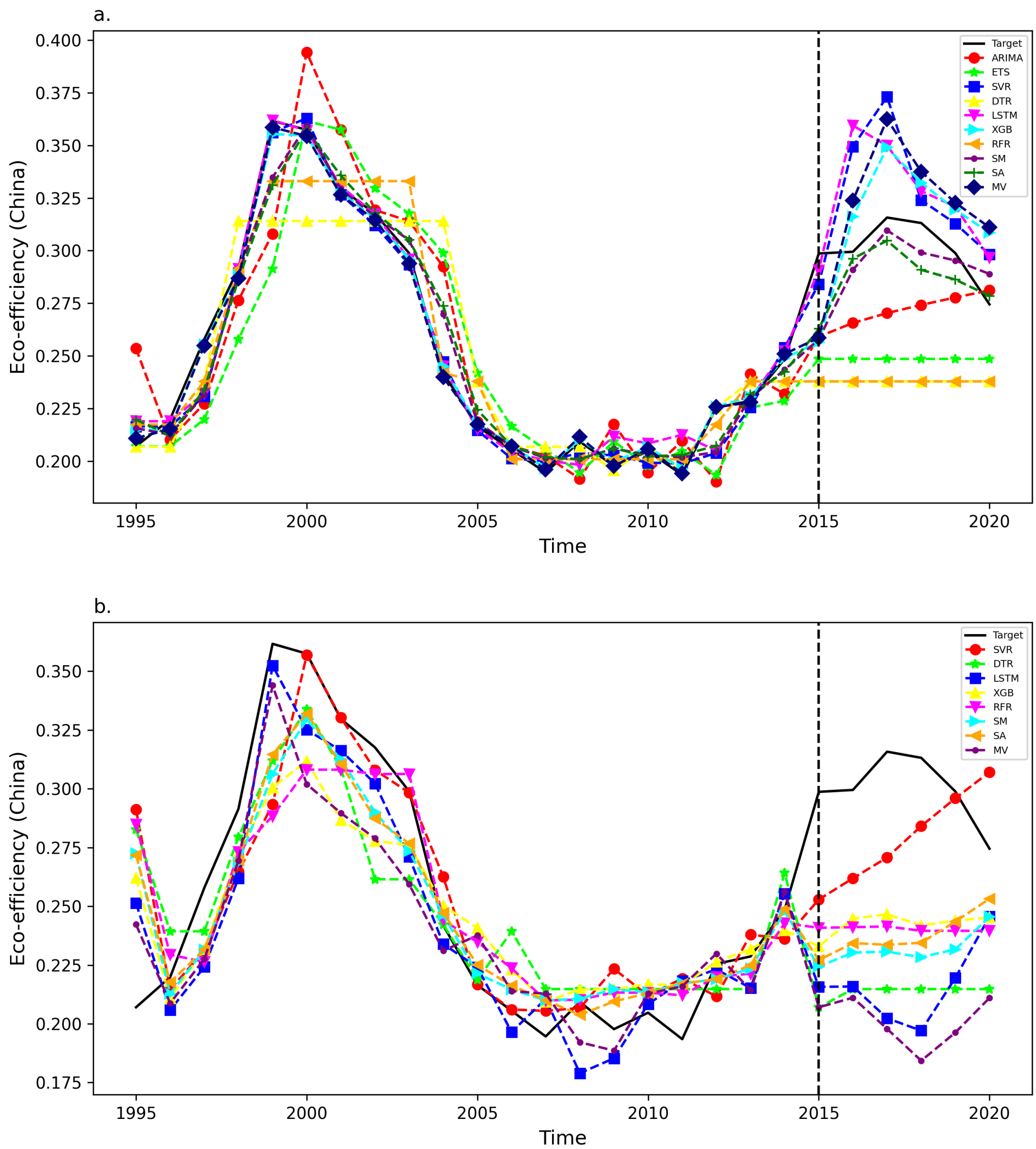


Figure 21 – Forecasts for Germany (G18) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

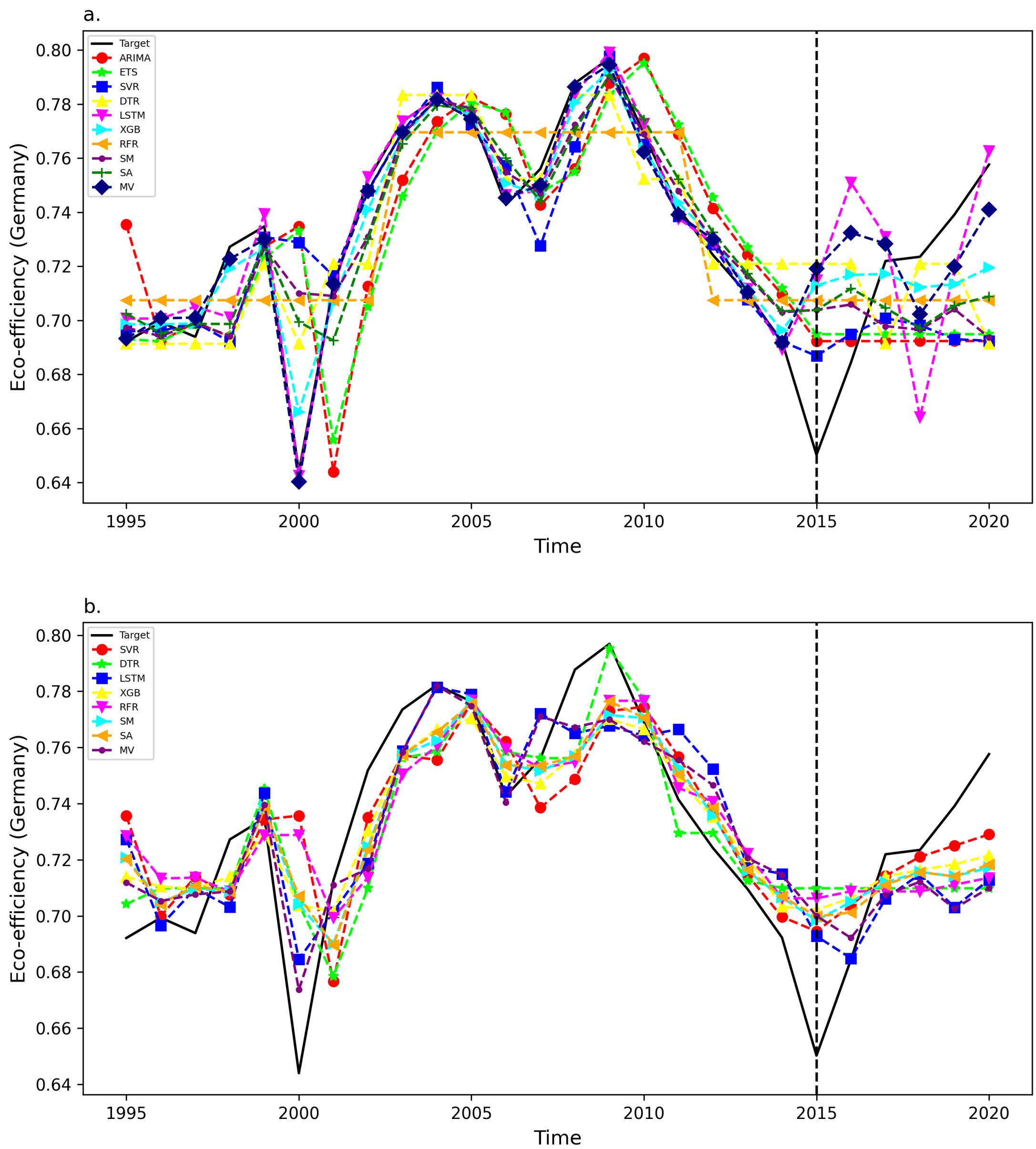


Figure 22 – Forecasts for India (G18) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

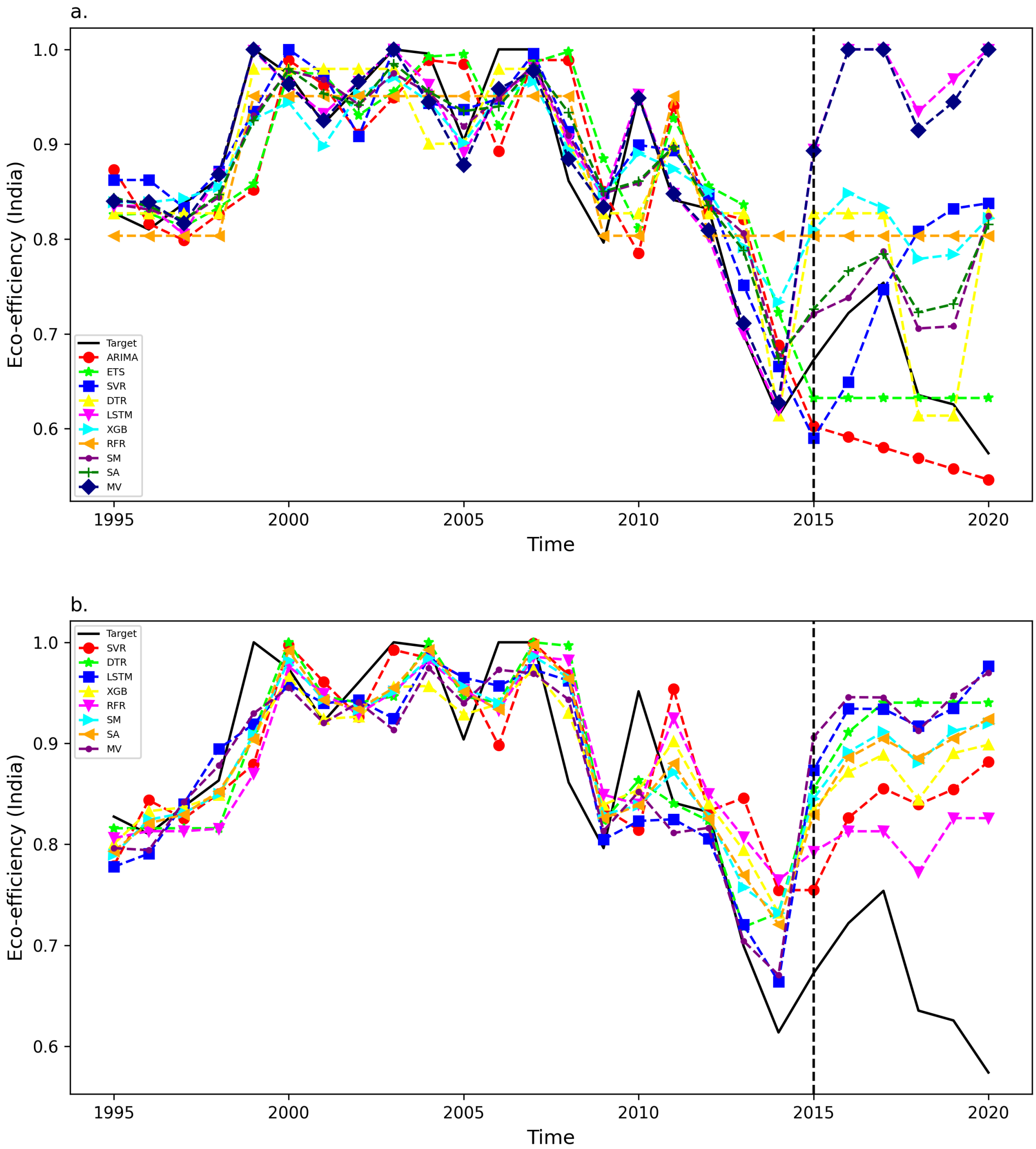


Figure 23 – Forecasts for Indonesia (G18) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

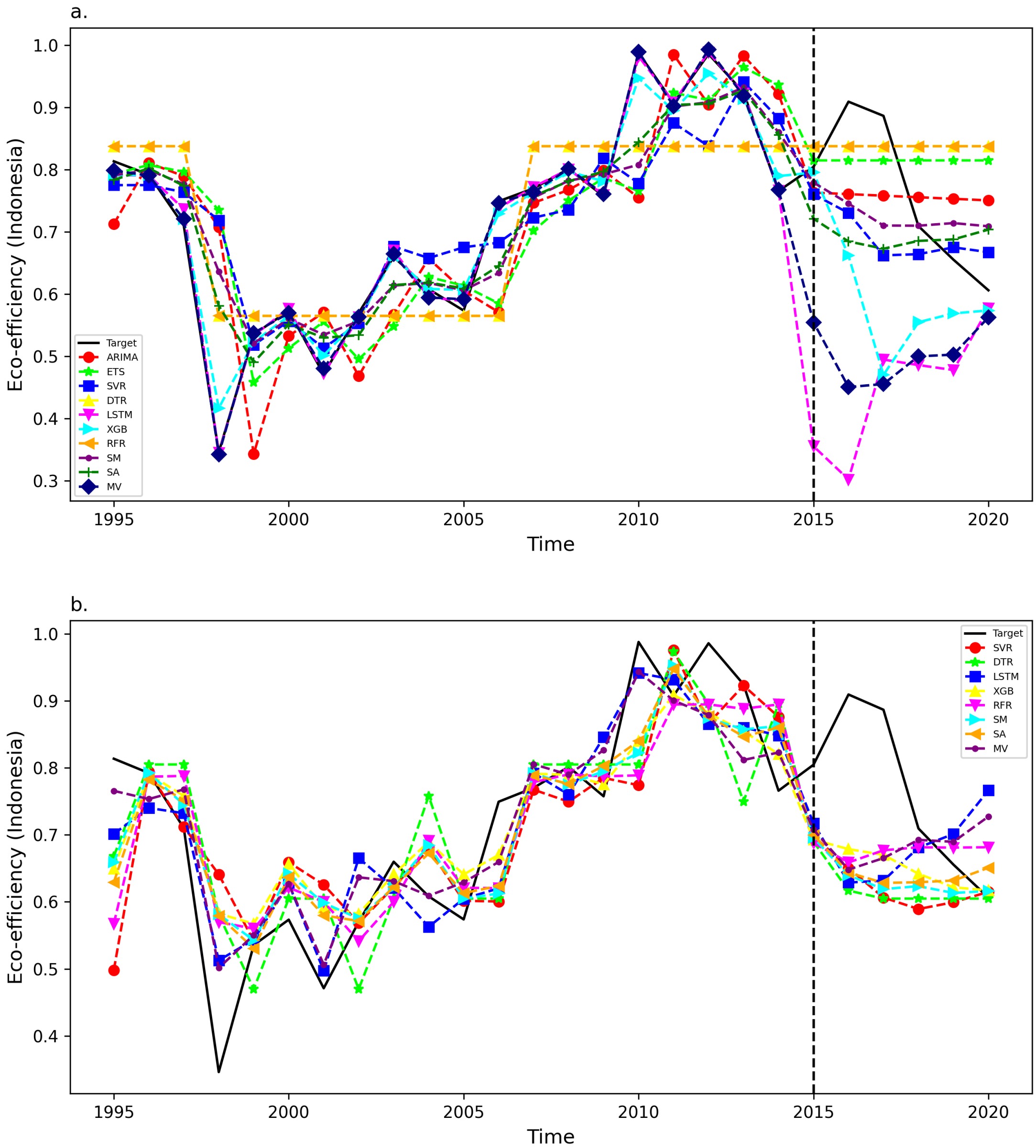


Figure 24 – Forecasts for South Korea (G18) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

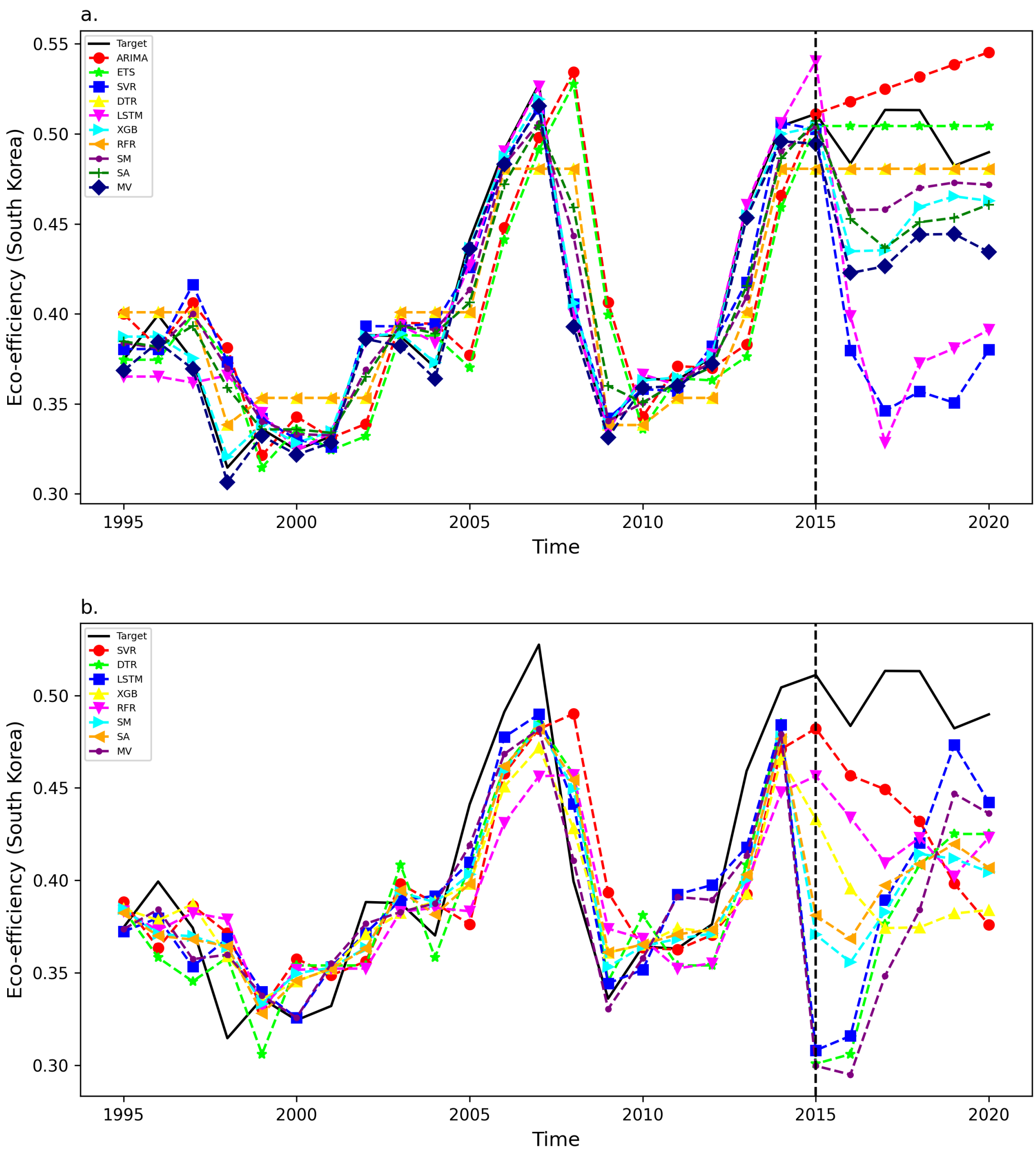


Figure 25 – Forecasts for Mexico (G18) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

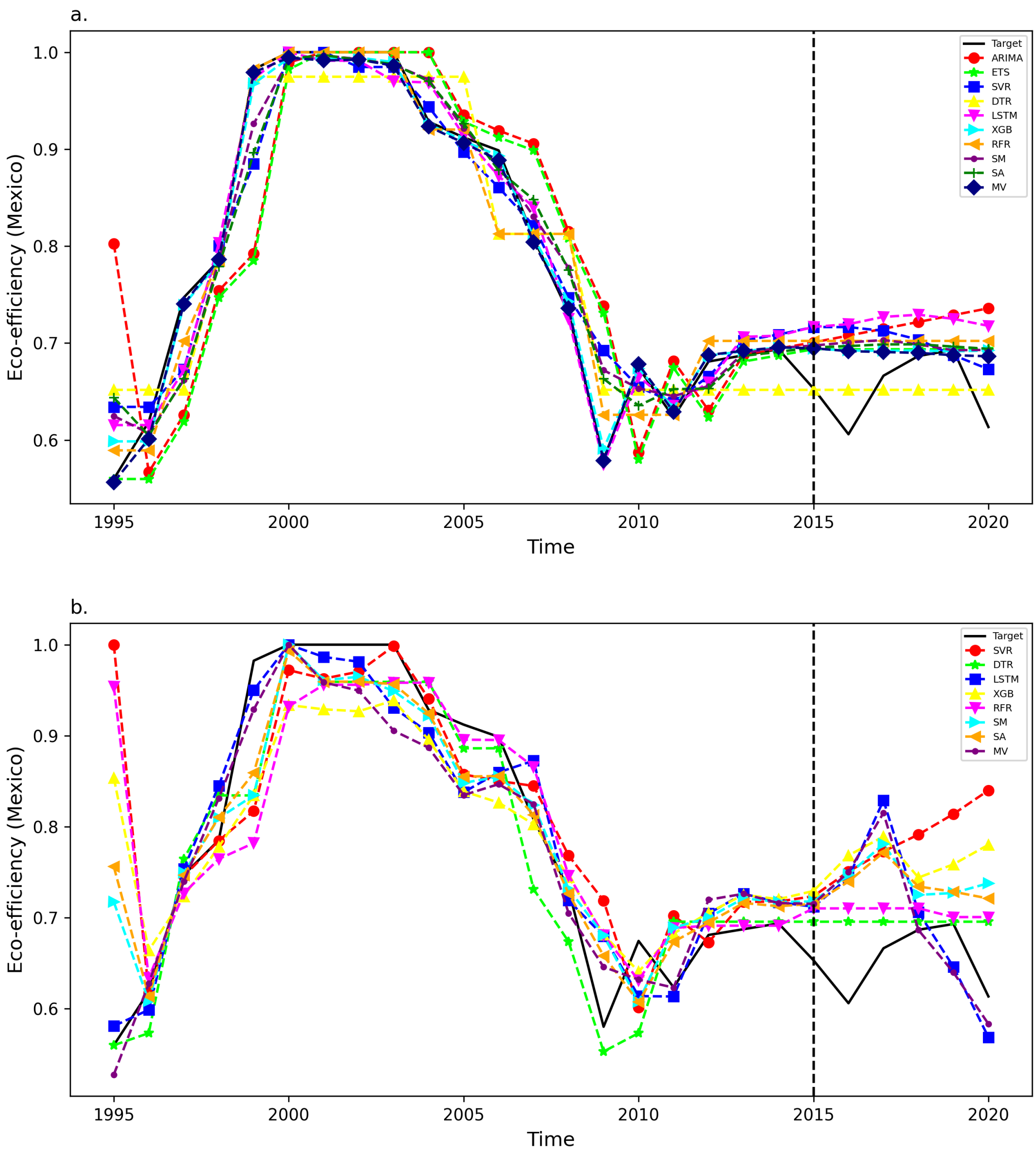


Figure 26 – Forecasts for South Africa (G18) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

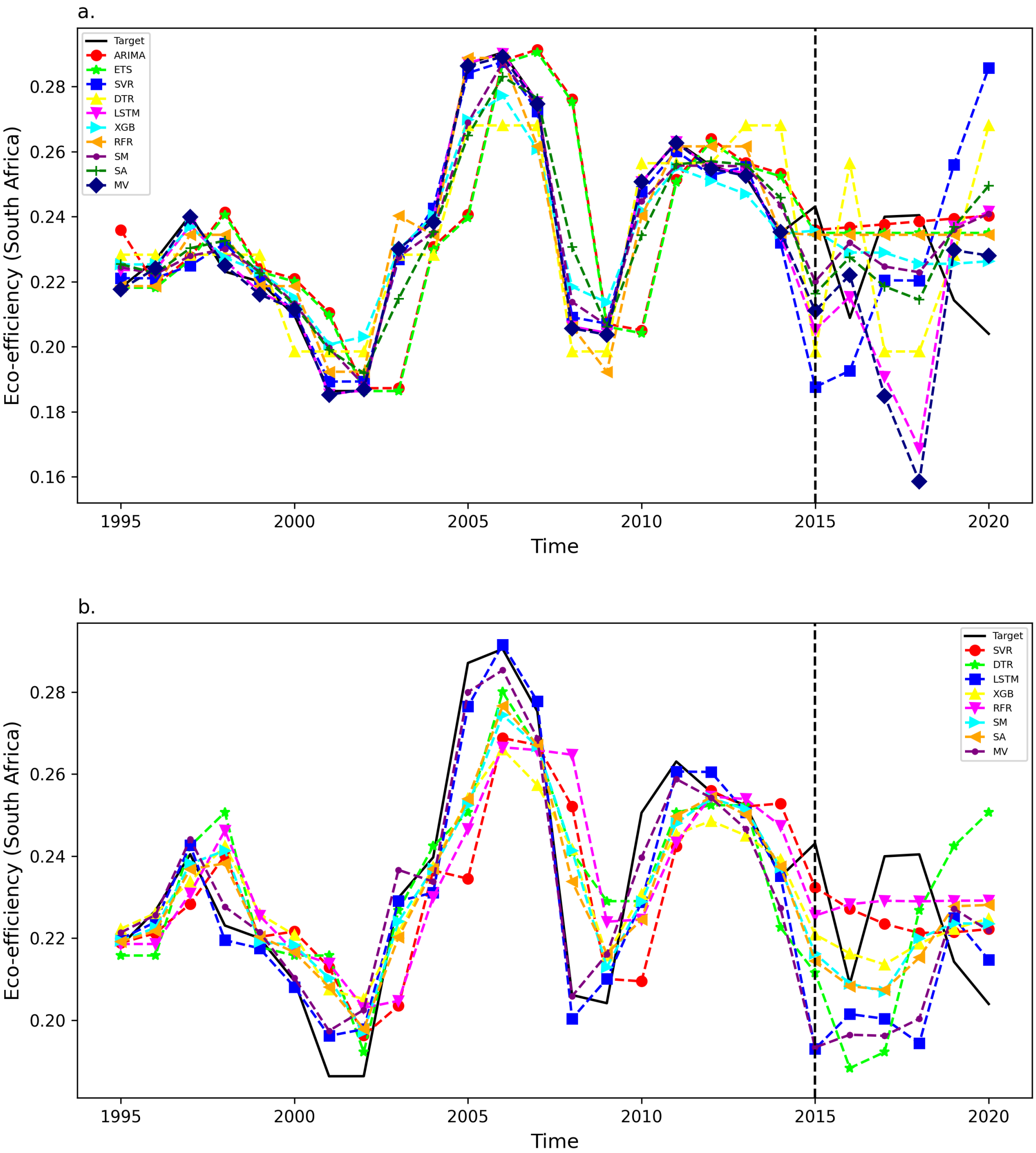


Figure 27 – Forecasts for Turkey (G18) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)

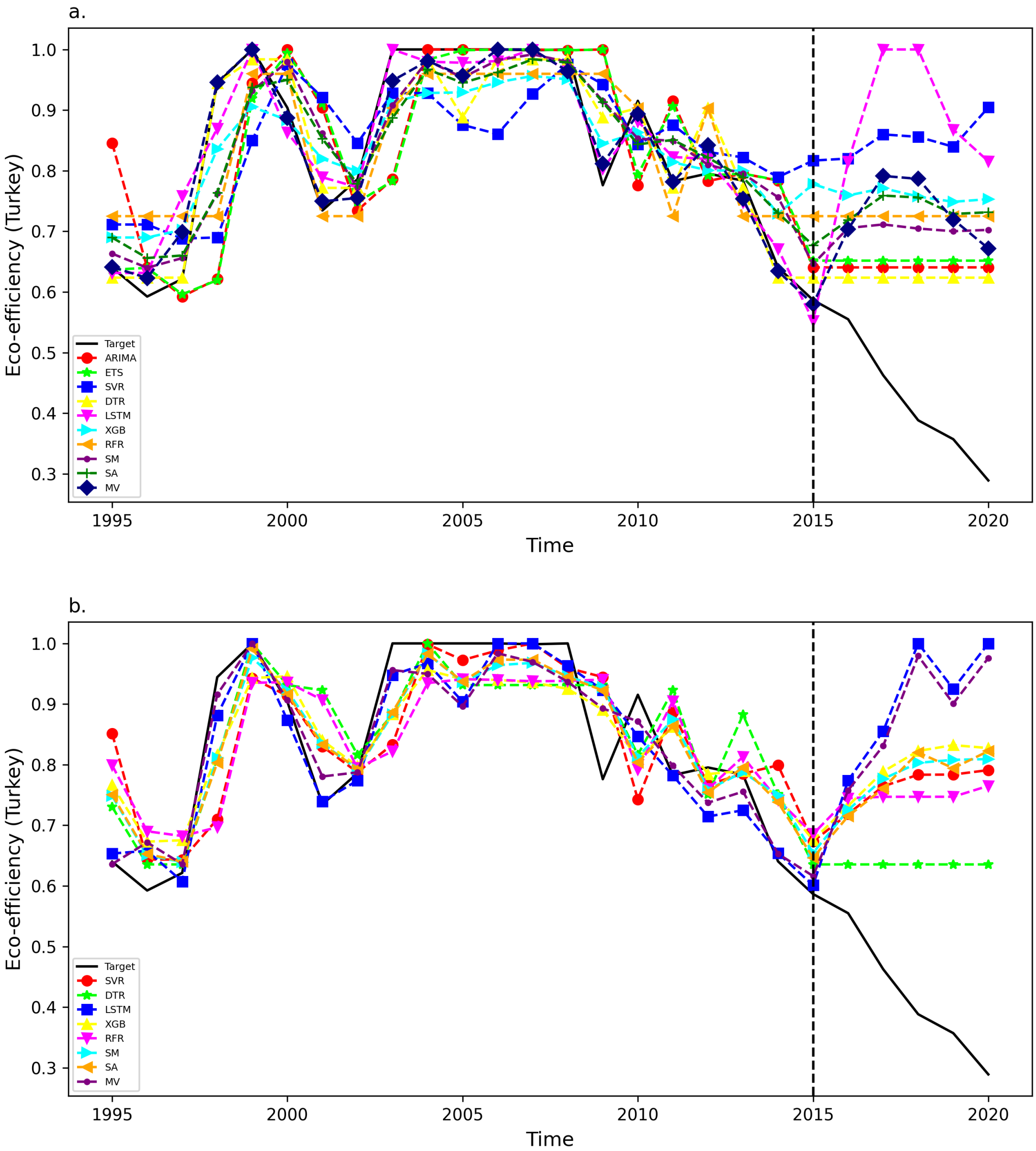
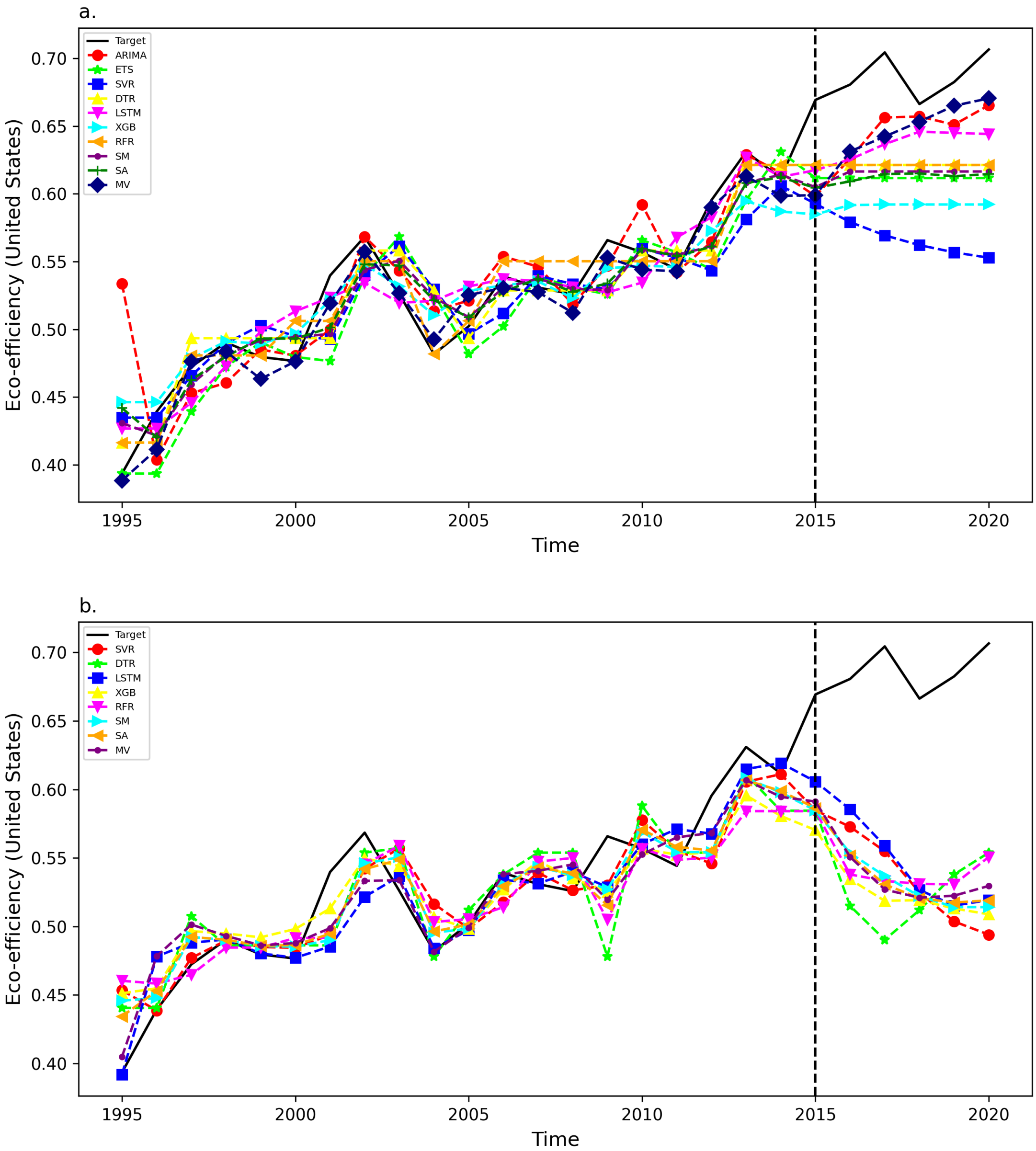


Figure 28 – Forecasts for United States (G18) considering approaches a. individual and b. pooled (the vertical dotted line indicates the start of the test)



APPENDIX D – ECO-EFFICIENCY PROJECTION

Table 31 – Eco-efficiency projection 6-years-ahead for Mercosul (2021-2026)

Time	ARG	BOL	PRY	URY
2021	0.811	0.824	0.481	0.841
2022	1.000	0.824	0.489	0.890
2023	1.000	0.824	0.498	0.897
2024	0.943	0.824	0.506	0.798
2025	0.860	0.824	0.514	0.781
2026	0.756	0.824	0.522	0.773

Table 32 – Eco-efficiency projection 6-years-ahead for BRICS (2021-2026)

Time	CHN	IND	RUS	ZAF
2021	0.424	0.923	0.378	1.000
2022	0.424	0.928	0.365	1.000
2023	0.424	0.945	0.364	1.000
2024	0.424	0.960	0.367	1.000
2025	0.424	0.976	0.373	1.000
2026	0.424	0.991	0.382	1.000

Table 33 – Eco-efficiency projection 6-years-ahead for G18 (2021-2026)

Time	ARG	AUS	CHN	DEU	IND	IDN	KOR	MEX	ZAF	TUR	USA
2021	0.328	1.000	0.274	0.695	0.632	0.565	0.504	0.652	0.208	0.944	0.685
2022	0.318	1.000	0.268	0.704	0.632	0.578	0.504	0.652	0.224	0.984	0.649
2023	0.308	1.000	0.266	0.714	0.632	0.595	0.504	0.652	0.234	0.984	0.624
2024	0.298	1.000	0.268	0.721	0.632	0.586	0.504	0.652	0.244	0.984	0.605
2025	0.289	1.000	0.273	0.725	0.632	0.613	0.504	0.652	0.234	0.984	0.593
2026	0.279	1.000	0.281	0.729	0.632	0.699	0.504	0.652	0.228	0.984	0.584

APPENDIX E – SLACK VARIABLES

Table 34 – Average (%) of slack variables for eco-efficiency in Mercosul (1995-2020)

Country	Arable land	Fixed capital	Labor force	Primary energy
Argentina	-53.86	-20.24	-22.67	-41.07
Bolivia	-47.39	-63.51	-62.83	-32.46
Paraguay	-73.92	-66.10	-63.10	-74.89
Uruguay	-49.86	-27.86	-33.75	-34.44

Table 35 – Average (%) of slack variables for eco-efficiency in BRICS (1995-2020)

Country	Arable land	Fixed capital	Labor force	Primary energy
China	-73.91	-81.99	-67.50	-67.83
India	-41.08	-35.05	-28.23	-5.23
Russian Federation	-74.83	-72.77	-71.40	-91.36
South Africa	-75.55	-52.10	-49.19	-75.75

Table 36 – Average (%) of slack variables for eco-efficiency in G18 (1995-2020)

Country	Arable land	Fixed capital	Labor force	Primary energy
Argentina	-57.92	-59.35	-66.03	-55.21
Australia	-6.84	-19.84	-20.00	-12.48
China	-78.62	-89.85	-84.67	-73.82
Germany	-39.44	-27.03	-28.46	-32.16
India	-79.66	-73.90	-66.60	-16.65
Indonesia	-63.40	-81.60	-79.27	-27.40
South Korea	-61.38	-66.31	-58.24	-66.89
Mexico	-27.78	-45.95	-39.52	-23.73
South Africa	-78.27	-77.64	-80.17	-76.82
Turkey	-54.11	-47.19	-32.02	-24.79
United States	-44.91	-44.31	-46.42	-72.47